Heterogeneous Intermediaries in a Production Economy [¶]

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Abstract

We study a general equilibrium model in a continuous-time production economy with heterogeneous agents and Epstein–Zin preferences in which financial intermediaries face Value-at-Risk constraints. Intermediaries are less risk-averse and more productive than households. We find that restrictions on their portfolios significantly dampen investment, reduce aggregate productivity, and lower economic growth. Log-utility specifications—commonly used for tractability—severely understate these effects and may even suggest that restrictions are irrelevant. Extending the model to include heterogeneous intermediaries, we find that the main mechanisms remain intact: restrictions binding for the most productive intermediaries continue to depress growth, even if less productive intermediaries can partially absorb risk. Our results highlight the importance of preference specifications and intermediary heterogeneity for understanding the macroeconomic consequences of financial regulation and for asset pricing in production economies.

Keywords: Asset Pricing, Intermediary Asset Pricing, Production Economy, Restriction

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1 Introduction

Understanding how frictions in the risk-taking capacity of financial intermediaries affect asset prices has become a central question in asset pricing. The intermediary asset pricing literature highlights how intermediaries' marginal valuations influence equilibrium returns across markets. When intermediaries face portfolio constraints — such as portfolio volatility bounds or capital limitations — risk premia, risk-free rates, and asset allocations adjust accordingly. These restrictions can also alter the real side of the economy. While most theoretical papers explore these effects in endowment economies, relatively few examine their implications in production economies where capital accumulation and consumption are endogenous. Moreover, these papers often rely on log utility to simplify model solutions. Our paper fills this gap by analyzing the real effects of intermediary portfolio constraints in a continuous-time production economy with heterogeneous agents featuring Epstein-Zin preferences.

We rely on a production-based general equilibrium asset pricing model with multiple investor groups. In the baseline model with homogeneous intermediaries, we consider financial intermediaries and households. All investors have Epstein-Zin preferences with different risk aversion levels and identical intertemporal elasticity of substitution. The intertemporal elasticity of substitution is assumed to be greater than one and households are more risk-averse than intermediaries. In addition, we assume that the investors differ in terms of their productivity (consumption good that is produced per unit of capital owned by the respective investor). Intermediaries have access to a large set of investment opportunities, including illiquid assets like infrastructure investments, and as a result have the highest productivity. Households, by contrast, are the most restricted in terms of their access to real investment opportunities and thus have the lowest productivity. We extend the model to allow for heterogeneous intermediaries, which comprise less risk-averse funds and more risk-averse banks; households are still the most risk-averse investors.

We assume that the most productive (and least risk-averse) financial intermediaries are subject to a Value-at-Risk constraint which limits the risk of their portfolio. This restriction builds on Kargar (2021), who studies this constraint in an endowment economy. In a production economy, this restriction not only affects financial quantities such as returns, but also has real implications. In particular, we will focus on the implications for long-run economic growth.

Our paper makes several contributions to the literature. First, we analyze the real implications of financial restrictions in a production-based asset pricing model with Epstein-Zin preferences. When the VaR-constraint binds, the demand for risky investments drops, which induces the price of capital and therefore the growth rate to decline.

Second, we show that the assumption of log preferences, which are analytically more tractable, dramatically underestimates the real implications of restrictions. Under log utility, the restriction has only minor effect on growth. The use of log preferences thus may lead to misleading conclusions.

Third, we analyze the mechanism underlying the difference in real effects between Epstein–Zin preferences and log utility. Moving from Epstein–Zin preferences to log utility simultaneously eliminates heterogeneity in risk aversion across investors, lowers aggregate risk aversion to one, and restricts the intertemporal elasticity of substitution to one. The equalization and reduction in risk aversion compresses differences in portfolio shares across investors, reducing leverage by the most productive intermediaries and lowering aggregate growth even in the absence of constraints. Moreover, with a unit intertemporal elasticity of substitution, consumption rates become constant and identical across agents, implying that intermediaries no longer save more than households and that their wealth share grows more slowly over time. As a result, financial constraints bind in an economy that is already less levered and less growth-oriented, substantially dampening the real effects of intermediary restrictions relative to the Epstein–Zin benchmark.

Fourth, we show that allowing for heterogeneity among financial intermediaries does not overturn our main qualitative results, but sharpens the contrast between Epstein-Zin and logarithmic preferences. We extend the model to include less risk-averse, highly productive funds and more risk-averse, less productive banks, while households remain the most risk-averse investors. Under Epstein–Zin preferences, the portfolio constraint binds primarily for the most productive intermediaries, resulting in stronger distortions to investment and a larger decline in aggregate growth. By contrast, in an economy with three investor types and logarithmic preferences, portfolio constraints have no real effects, as the equalization of risk aversion and the unit intertemporal elasticity of substitution eliminate the channels through which financial restrictions affect growth.

We contribute to several strands of the literature. The research on intermediary asset pricing focuses on the impact of intermediaries on asset pricing moments. Intermediaries are subject to frictions or restrictions rather than being a *veil* passing on households' preferences as is assumed in previous literature. The theoretical foundations of intermediary asset pricing trace back to He and Krishnamurthy (2012, 2013). These papers study an endowment economy with households and financial intermediaries and show that a binding equity capital constraint explains changes in risk premia during crises. Kargar (2021) studies an intermediary asset pricing model with heterogeneous intermediaries and a household in an endowment economy. He shows that the reaction of intermediaries to consumption shocks differs depending on their risk aversion. This explains different behaviors of various groups of intermediaries that have also been observed empirically as stated below.

The second strand we contribute to is literature on production-based asset pricing models. It was started on several publications by Cochrane (1991, 1996) where he develops the main framework and tests its results empirically. A large subsequent literature extends this framework. Campbell (2018, chapter 7) gives a detailed explanation of the mechanisms in production based models and includes an overview of different models.

Several papers study theoretical intermediary asset pricing in the setting of a production economy, analyzing the impact of intermediation and frictions on investment and output. Brunnermeier and Sannikov (2014) allow for two investors who have linear preferences but differ with respect to output factors (productivities) and depreciation rates. Their model shows that highly nonlinear amplification effects may lead to an unstable economy with volatile crisis episodes. Gertler and Kiyotaki (2015) use a discrete-time model with liquidity mismatch and bank runs. If a bank run occurs, it induces a huge decrease in economic activity. Even if agents anticipate a bank run without it actually happening there is a negative impact. He and Krishnamurthy (2019) study a model with two agents with log-utility and equity capital constraint. They are able to explain stylized facts from the (onset of) the global financial crisis with only intermediary capital shocks. Their paper offers a way to map macro-stress tests to the probability of systemic states. Elenev, Landvoigt, and Van Nieuwerburgh (2021) study a model with a government sector which is able to explain stylized facts related to crises and assess the impact of capital restrictions on welfare. Maxted (2023) studies a model with diagnostic expectations and shows the role of sentiment. Finally, Krishnamurthy and Li (2024) consider a model with two log-utility investors and sentiment. Their mechanism manages to match data on credit spreads, equity prices, credit, and output during the onset and aftermath of a crisis.

Intermediary asset pricing has been tested empirically by Adrian, Etula, and Muir (2014); He, Kelly, and Manela (2017) and many more. Adrian, Etula, and Muir (2014) construct an empirical intermediary stochastic discount factor based on shocks to broker-dealers' leverage. Their factor captures deteriorating funding conditions that come with intermediary deleveraging; bad states are those states with low leverage and a high marginal utility of wealth. He, Kelly, and Manela (2017) follow a similar approach and also construct an intermediary risk factor. They focus on primary dealers as intermediaries and find that capital shocks are priced in the cross-section of asset classes. Other papers worth mentioning are Haddad and Muir (2021); Baron and Muir (2022); Du, Hébert, and Huber (2023); Bauer, Bernanke, and Milstein (2023).

Finally, our paper is also linked to the literature on asset pricing models with heterogeneous investors. References in this area include Dumas (1989), Wang (1996), Gârleanu and Pedersen (2011), Bhamra and Uppal (2013), and Gârleanu and Panageas (2015). They show that heterogeneity can have a significant impact on risk premia, asset prices, and further asset pricing moments.

The remainder of this paper is organized as follows. Section 2 explains the model and its solution. Derivations and additional details on the solution algorithm can be found in the Appendix. Section 3 presents the results, focusing on the role of the VaR constraint and comparing the Epstein-Zin and log-utility economies. Section 4 concludes.

2 Model Setup

We have two separate model set-ups. Their main difference is the number of representative agents. In the first set-up, there are two agents, banks and households, referred two as Type "a" and Type "c" agents. The second model consists of three agents, the banking sector is divided in funds (Type "a" agents) and banks (Type "b" agents). This chapter begins by explaining the set-up of the two-agent model and then shows the differences that come with the third agent.

Type a agents are the least risk-averse and type c agents are the most risk-averse. We use Epstein-Zin utility and keep the intertemporal elasticity of substitution constant across the agents.

Agents derive utility from consumption. The consumption good is produced by capital, in which agents must invest a positive fraction of their wealth. It is not possible to short real production units. Agents can also engage in risk-less borrowing and lending among themselves. Furthermore, there can be VaR-restrictions on the volatility of the portfolio return of banks. Apart from risk aversion, agents are also heterogeneous in their returns to capital: Type a agents have the highest return on capital as they are able to invest in the most lucrative projects, while type c agents are not as skilled and invest in less lucrative investment opportunities. In the following, each individual aspect of the model will be portrayed in detail.

2.1 Agents

The economy is populated by two agents indexed by

$$i \in \{a, c\}, \tag{1}$$

where type "a" agents correspond to *Banks* or *intermediaries* and type "c" agents to *Households*. Furthermore, agents exhibit the following ordering for the *risk aversion* parameter γ such that

$$\gamma_a < \gamma_c. \tag{2}$$

All agents exhibit the same *intertemporal elasticity of substitution*, i.e.

$$\psi_a = \psi_c = \psi. \tag{3}$$

Due to (2) and (3), agents feature *Epstein-Zin* preferences[5].

On top of that, each agent has a certain skill level such that type "a" investors are the most savy who are able to finance the projects with the highest return on capital. We characterize this by the constants

$$A_a > A_c, \tag{4}$$

that correspond to the *productivity* of each agent's capital holdings (see Section 2.2).

State Variables. Each agent is fully characterised by the joint *wealth* levels $W_{a,t}$ and $W_{c,t}$. Agents jointly interact through market clearing so that they need to know the decisions of all other agents as via market clearing this will affect their own decisions. In this production-based environment, wealth corresponds to the to be determined value of capital (see (11)).

Policy Functions. Each agent has three policy functions. The first policy is *consumption* $C_{i,t} \geq 0$ which is the sole source from which the agent derives utility.

The second policy is the *portfolio weight*

$$\pi_{i,t} \geq 0, \tag{5}$$

that allocates a share $\pi_{i,t}$ of the agent's wealth to the risky asset (capital) that earns the to be determined *per unit* of capital return $dr^K_{i,t}$ (see (16)), and the remaining share $(1 - \pi_{i,t})$ to a riskfree bond that earns the riskless rate r_t. Notice that short-selling real assets such as capital is not possible which is why it is

[5] For our calibration it holds that $\gamma_i > \psi_i \ \forall i$. See Table 2.

excluded through the restriction $\pi_{i,t} \geq 0$. On the other hand, borrowing the riskless asset is permitted.

The third policy is the investment rate $\iota_{i,t} \geq 0$ which is determined in Section 2.2 (see (13)).

Optimisation Problem. Each agent maximizes its lifetime-utility of consumption so that it solves the following optimization problem

$$V_{i,t} = \max_{\{C_{i,s},\pi_{i,s},\iota_{i,s}\}_{s\in[t,\infty)}} \mathbb{E}_t \left[\int_t^\infty f_i\big(C_{i,s}, V_{i,s}\big)\, ds \right] \qquad \text{s.t. } \frac{dW_{i,t}}{W_{i,t}} = \pi_{i,t} dr^K_{i,t} + (1-\pi_{i,t}) r_t dt - c_{i,t} dt, \tag{6}$$

where $c_{i,t}$ is the individual consumption-wealth ratio[6], i.e. $\frac{C_{i,t}}{W_{i,t}}$, and ρ_i is the discount rate.

To ensure a stationary solution and the persistence of heterogeneity over time, we consider an overlapping-generations set-up. κ describes the rate at which agents are born and die. The distribution of newborn investors is given by $\bar{x}$. This means that at each point in time, a fraction of the intermediaries die such that their wealth share decreases by κx_t and there are so many newborn intermediaries that their wealth share increases by $\kappa \bar{x}$. $1-\bar{x}$ type "c" agents are newborn.

Note that the aggregator function is given by

$$f_i\big(C_i, V_i\big) = \frac{1-\gamma_i}{1-\frac{1}{\psi_i}} \cdot V_i \cdot \left[\left(\frac{C_i}{\big[(1-\gamma_i)V_i\big]^{\frac{1}{1-\gamma_i}}} \right)^{1-\frac{1}{\psi_i}} - (\rho_i + \kappa) \right]. \tag{7}$$

Reformulating the State Variables. We define the following wealth share

$$x_t \equiv \frac{W_{a,t}}{W_{a,t} + W_{c,t}} \tag{8}$$

such that x_t is the overall wealth share of the *bank* of the overall economy.

The goal of this reformulation is that it eases interpretation, as the share of the overall economy is a metric that can be used to compare the model to the real world, and it provides a natural domain

[6]This quantity is easier to interpret than raw consumption. For a given wealth level (state), maximising the consumption-wealth ratio is equivalent to maximising consumption.

restriction inside the unit interval for each state variable. This will aide in finding a stationary solution.

However, the re-formulation incurs that the dynamics of the state variable become more complicated. These are given in (20) after the whole model has been laid out.

2.2 Production Sector

In this Section we explain the production side of the economy.

Capital. Firstly, *aggregate* capital K_t evolves according to

$$dK_t = (\iota_t - \delta)K_t dt + \sigma_K K_t dZ_t, \tag{9}$$

where ι_t is the to be determined *aggregate* investment rate ($\iota_t K_t$ is the level of aggregate investment), δ is the constant depreciation rate, σ_K is the volatility and Z_t is a standard Wiener process.

Price of Capital. The price for installed capital is p_t. We denote the dynamics of this price of capital by

$$\frac{dp_t}{p_t} = \mu_{p,t} dt + \sigma_{p,t} dZ_t \tag{10}$$

where p_t, $\mu_{p,t}$ and $\sigma_{p,t}$ are all functions of the state variable x_t. The to be determined coefficients $\mu_{p,t}$ and $\sigma_{p,t}$ are given in (21).

For each agent it then holds that $K_{i,t} = \pi_{i,t} \frac{W_{i,t}}{p_t}$ where $K_{i,t}$ is the units of capital held by agent i. Thus, the price for capital and the wealth level jointly determine how many units of capital each agent holds.

Lastly, since in this production-based setting aggregate wealth corresponds to the aggregate value of capital, it holds that

$$W_{a,t} + W_{c,t} = p_t K_t. \tag{11}$$

Investment Rate. Each agent wants to maximize their profit of investment. Since investment into capital is financed by foregoing consumption (see (19)) and the price of the consumption good is normalised to 1, the profit from investing into capital is given by

$$\max_{\iota_{i,t}} p_t \iota_{i,t} - \iota_{i,t} - \Phi(\iota_{i,t}) \qquad \text{with} \qquad \Phi(\iota_{i,t}) = \frac{\chi}{2}\big(\iota_{i,t} - \delta\big)^2, \tag{12}$$

where we additionally assume quadratic capital adjustment costs. The parameter χ is the investment adjustment coefficient which is used to calibrate the model.

Since the price and depreciation rate of capital is the same for all agents, this entails that the optimal investment rate is determined by $\iota^*_{i,t} = \frac{p_t - 1}{\chi} + \delta \ \forall i$. We conclude that the agent specific investment rates are identical so that the *aggregate* investment rate is trivially given by[7]

$$\iota^*_t = \frac{p_t - 1}{\chi} + \delta. \tag{13}$$

Output Good. Capital is used to produce the final output good which finances both consumption and re-investment to capital. Each investor produces output according to the following technology $Y_{i,t} = A_i K_{i,t}$. Thus, the *aggregate* output good Y_t is produced according to[8]

$$Y_t = \overline{A}_t K_t, \tag{14}$$

[7] Since $\sum_i K_{i,t}\iota^*_{i,t} = \sum_i K_{i,t}\iota^*_t = K_t \iota^*_t$ which is the level of investment.

[8] Consider investor a. Then, $Y_{a,t} = A_a K_{a,t} = A_a \pi_{a,t} \frac{W_{a,t}}{p_t} = A_a \pi_{a,t} \frac{W_{a,t}}{p_t K_t} K_t \overset{(8)\&(11)}{=} A_a \pi_{a,t} x_t y_t K_t$. $\sum_i Y_{i,t}$ yields the result.

where $\overline{A}_t$ is the aggregate productivity given by

$$\overline{A}_t = \pi_{a,t} x_t A_a + \pi_{c,t}(1 - x_t) A_c. \tag{15}$$

Hence, the higher the share of type "a" agents in the economy and the more they invest in capital, the higher the *aggregate* productivity and thus the more of the *aggregate* output good is produced, which in turn implies that consumption and thus utility is higher.

Return on Capital. The return *per unit* of capital, $dr_{i,t}^K$, is determined by the dividend yield and the evolution of the value of capital. Since each investor exhibits different returns to capital, the dividend yield will differ across investors.

The evolution of the value of capital is given by the Ito product rule using (9) and (10) such that

$$\frac{d(p_t K_t)}{p_t K_t} = (\iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t})dt + (\sigma_K + \sigma_{p,t})dZ_t.$$

An agent's dividend yield per unit of capital is given by

$$\frac{\big(A_i - \iota_t^* - \Phi(\iota_t^*)\big) K_{i,t}}{p_t K_{i,t}}$$

where the numerator is the value of the consumption good and the denominator is the value of the agent's capital. Thus, the overall *per unit* return of capital held by agent i is given by

$$dr_{i,t}^K = \left(\frac{A_i - \iota_t^* - \Phi(\iota_t^*)}{p_t} + \iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t} \right) dt + \big(\sigma_K + \sigma_{p,t}\big) dZ_t. \tag{16}$$

2.3 Market Clearing

In this Section the market clearing conditions will be shown.

Capital Market Clearing. Capital market clearing implies that the sum of shares of wealth invested into capital is equal to the aggregate value of capital[9]. Dividing by aggregate wealth $\sum_i W_{i,t}$ and using (11), we can re-write this in terms of the state variables x_t and y_t such that capital market clearing is given by

$$\pi_{a,t}x_t + \pi_{c,t}(1 - x_t) = 1 \tag{17}$$

Bond Market Clearing. Since the riskless asset is in zero net supply, bond market clearing is simply given by[10]

$$(1 - \pi_{a,t})x_t + (1 - \pi_{c,t})(1 - x_t) = 0. \tag{18}$$

Notice that if the capital market is cleared, i.e. (17) holds, then (18) holds automatically since $x_t + (1 - x_t) = 1$. In other words, one of these market clearing conditions is redundant.

Goods Market Clearing. Since consumption and investment are financed from the output good, this is given by[11]

$$\overline{A}_t = p_t\big[c_{a,t}x_t + c_{c,t}(1 - x_t)\big] + \iota^* + \Phi(\iota^*). \tag{19}$$

Notice the real nature of the capital adjustment costs since they must be financed from the output good.

2.4 HJB

In this Section the HJB equation for the problem (6) under the reduced state variable x_t will be derived.

To do so, the state dynamics require a derivation. From the definition (8) using (6) and (16) it

[9] I.e. $\pi_{a,t}W_{a,t} + \pi_{c,t}W_{c,t} = p_tK_t$.

[10] This follows from $(1 - \pi_{a,t})W_{a,t} + (1 - \pi_{c,t})W_{c,t} = 0$ and dividing by $\sum_i W_{i,t}$ while using (8).

[11] This follows from $Y_t = C_{a,t} + C_{c,t} + \big[\iota_t^* + \Phi(\iota_t^*)\big]K_t$. Insert (14) for output, express $C_{i,t} = c_{i,t}W_{i,t}$ and extend it with p_tK_t. Then, divide by K_t, use (11) in the denominator and insert the wealth shares (8) to yield the result.

follows that

$$
\begin{aligned}
dx_t =& \kappa(\bar{x} - x_t)dt && (20)\\
&+ x_t(1-x_t)\Bigg[\big(\pi_{a,t} - \pi_{c,t}\big)\big(\iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t} - r_t\big)\\
&\quad - \big(\pi_{a,t} - \pi_{c,t}\big)\big(x_t \pi_{a,t} + (1-x_t)\pi_{c,t}\big)\big(\sigma_K + \sigma_{p,t}\big)^2\\
&\quad + \left(\pi_{a,t}\frac{A_a - \iota_t^* - \Phi(\iota_t^*)}{p_t} - \pi_{c,t}\frac{A_c - \iota_t^* - \Phi(\iota_t^*)}{p_t}\right) + c_{c,t} - c_{a,t}\Bigg]dt\\
&+ \Big[x_t(1-x_t)(\pi_{a,t} - \pi_{c,t})(\sigma_K + \sigma_{p,t})\Big]dZ_t.
\end{aligned}
$$

However, in x_t the drift ($\mu_{p,t}$) and diffusion ($\sigma_{p,t}$) coefficient of the price p_t is still unknown. As everything, apart from the constants, is a function of the state variable x_t, we use this fact to obtain the following for $\mu_{p,t}$ and $\sigma_{p,t}$

$$
\frac{dp_t(x)}{p_t(x)} = \underbrace{\left(\frac{p_{t,x}}{p}\mu_{x,t} + 0.5\frac{p_{t,xx}}{p_t}\sigma_{x,t}^2\right)}_{=\mu_{p,t}} dt + \underbrace{\left(\frac{p_{t,x}}{p_t}\sigma_{x,t}\right)}_{=\sigma_{p,t}} dZ_t. \tag{21}
$$

At first this only shifts the problem to another layer since the partial derivatives of the price function are unknown. While this is true, this problem can be tackled by solving the model as a sequence of partial equilibria so that in each iteration a price function is guessed, the model is solved under this price function, and the price is updated so that market clearing conditions hold. This sequence of partial equilibria is solved until the general equilibrium is reached. Thus, for each partial equilibria $\mu_{p,t}$ and $\sigma_{p,t}$ are known.

Using (20) on (6), the HJB for each agent is given by the following where $G_{i,t} \equiv G_i(t, x_t)$ is the

Value Function of agent i (The exact derivation is in the appendix in section C.2)

$$\begin{aligned}\rho_i + \kappa =& \max_{\pi_{i,t}} \Bigg\{ G_{i,t} + (1-\tfrac{1}{\psi_i}) \left(r_t + \pi_{i,t} \left(\frac{A_i - \iota_t^* - \Phi(\iota_t^*)}{p_t} + \iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t} - r_t \right) - G_{i,t} \right) \\ & - \tfrac{1}{\psi_i} \frac{G_{i,t,x}}{G_{i,t}} \mu_{x,t} - \tfrac{1}{2}(1-\tfrac{1}{\psi_i}) \gamma_i \pi_{i,t}^2 (\sigma_K + \sigma_{p,t})^2 \\ & - \tfrac{1}{2} \tfrac{1}{\psi_i} \left((\tfrac{1-\gamma_i}{1-\psi_i} - 1) \frac{(G_{i,t,x})^2}{G_{i,t}^2} + \frac{G_{i,t,xx}}{G_{i,t}} \right) \sigma_{x,t}^2 \\ & - \frac{1-\gamma_i}{\psi_i} \frac{G_{i,t,x}}{G_{i,t}} \pi_{i,t} \sigma_{x,t} (\sigma_K + \sigma_{p,t}), \end{aligned} \tag{22}$$

and optimal investment is substituted from (13).

The first-order condition for the portfolio weights gives the following form:

$$\pi_{i,t} = \frac{1}{\gamma_i} \frac{\left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota - \delta + \mu_p + \sigma_K \sigma_t^p - r_t \right)}{(\sigma_K + \sigma_t^p)^2} + \frac{1}{\gamma_i} \frac{1-\gamma_i}{1-\psi_i} \left(\frac{G_{i,x}}{G_i} \frac{(\sigma_K + \sigma_t^p)\sigma_x}{(\sigma_K + \sigma_t^p)^2} \right) \tag{23}$$

The first term is the myopic demand for the capital which is the same for investors with logarithmic preferences. The second term is the hedging demand for changes in the state variables.

2.5 Value-at-Risk restriction

The intermediaries are subject to a Value-at-Risk constraint on their portfolio. Because we consider locally normally distributed return innovations, this constraint on instantaneous returns implies an upper bound on the volatility of the risky portfolio, i.e.,

$$\pi_{i,t}(\sigma_K + \sigma_{p,t}) \leq \bar{\sigma}_{FI} \tag{24}$$

We thus allow for the general case that the restriction does not bind at all, and that the restriction binds for type "a".

Table 1: Overview of Symbols

Symbol	Type	Description	Reference
γ_i	Constant	Agent's Risk Aversion	(2)(42)
ψ_i	Constant	Agent's Intertemporal Elasticity of Substitution	(3)
A_i	Constant	Agent's productivity in production function	(4)(43)
$\pi_{i,t}$	Function	Share of Wealth of agent i invested into Capital	(5)
$c_{i,t}$	Function	Agent's Consumption-Wealth Ratio	(6)
r_t	Function	Riskless Rate	(6)
ρ_i	Constant	Agent's Discount Rate	(7)
κ	Constant	Agents' birth and death rate	(7)
$\bar{x}$	Constant	Share of newborn intermediaries	
$\bar{y}$	Constant	Share of newborn type a agents within intermediaries	
x_t	Variable	Wealth Share	(8)
y_t	Variable	Wealth Share	(8)
δ	Constant	Depreciation Rate of Capital	(9)
σ_K	Constant	Volatility Scale of Capital	(9)
p_t	Function	Price of Capital	(10)
$\mu_{p,t}$	Function	Drift Coefficient of Price Function	(21)(49)
$\sigma_{p,t}$	Function	Diffusion Coefficient of Price Function	(21)(49)
ι_t	Function	Agents' Investment Rate	(13)
χ	Constant	Investment Adjustment coefficient	(12)
$\overline{A}_t$	Function	Aggregate Productivity	(15)(45)
$\bar{\sigma}_{FI}$	Constant	Upper bound on portfolio volatility	(24)

The arguments of all functions are the state variables x_t in the two-agent model and x_t and y_t in the three-agent model.

2.6 Changes in the three-agent model

The introduction of a third agent allows us to study heterogeneity within the financial sector. Additionally to the agents of Type "a" (intermediaries, in the three-agent version funds) and Type "c" (households) introduced in the previous sections, we introduce agents of Type "b", which we think to be banks. The risk-aversion of Type "b" is is calibrated to be between the risk-aversions of Types "a" and "c", the same holds for the productivity. Specifically, we assume

$$\gamma_a < \gamma_b < \gamma_c \quad \text{and} \quad A_a \geq A_b > A_c. \tag{25}$$

All changes that come from introducing the third agent are highlighted in the Appendix in section D.1. The main difference between the two model versions with two investors and three investors is the number of state variables. In the previously described version of the model, we had one state variable, the wealth share of the bank x_t, to reduce the model to. In the version with three agents, x_t describes the wealth share of the financial sector (Types "a" and "b" together) , i.e.,

$$x_t \equiv \frac{W_{a,t} + W_{b,t}}{W_{a,t} + W_{b,t} + W_{c,t}} \tag{26}$$

and the composition of the financial sector is described by

$$y_t \equiv \frac{W_{a,t}}{W_{a,t} + W_{b,t}}. \tag{27}$$

2.7 Logarithmic Utility

To analyze the impact of risk-aversion and elasticity of intertemporal substitution, we also solve the model for the specification that all agents have

$$\gamma_i = 1 \text{ and } \psi_i = 1. \tag{28}$$

This is equivalent to logarithmic utility. The calculations are shown in the Appendix in section F for the three agent model.

2.8 Solution algorithm

We have to solve the model numerically. Due to all equations being linear in K_t and a separation of wealth approach, we can restrict the set of state variables to x_t in the two-agent model and analogously to x_t and y_t in the three-agent model.[12] In other words, the Value Function G_i of each agents only depends on x_t (and y_t). Additionally, we need to guess a Price Function p_t as was explained in Equation (21) and

[12]This is also shown in the derivation of the HJB equation in the appendix.

(49). Details on the solution algorithm are given in the appendix in E.

3 Quantitative Results and Mechanisms

In this section, we discuss the results and implications of the different versions of the model. We are in particular interested in the real implications of the VaR-restriction which is imposed on the less risk-averse financial intermediary with the highest productivity. We therefore analyze the growth rates of consumption, output, and capital. Furthermore, we analyze the impact of the preference specification to assess whether the use of log preferences, which significantly eases computations, biases conclusions. To understand the mechanisms, we examine additional variables, such as the investment rate and average productivity, and consider the state-dependence of policy functions and economic variables. The welfare implications are assessed using certainty equivalent consumption.

3.1 Parameters

Table 2 shows the parameters used in the following. They are chosen to approximately match the growth rate in a benchmark model with a growth of 1.8%. We set the risk aversion to $\gamma_a = 2.5$ for funds (type "a"), $\gamma_b = 5.5$ for banks (type "b"), and $\gamma_c = 8.5$ for households (type "c"). Funds are thus the least risk-averse investors, while households are the most risk-averse ones. The intertemporal elasticity of substitution is 1.5 for all investors; the rate of time preference ρ is equal to 3% p.a.. The birth- and death-rate is $\kappa = 0.017$. The distribution of investors at birth is given by $\bar{x} = 0.05$ and, in case there are three investors, $\bar{y} = 0.20$. For the dynamics of capital, we set the depreciation rate $\delta = 0.1$, the capital adjustment costs have a multiplier of $\chi = 3$ and the volatility of capital is $\sigma_K = 0.12$. Finally, the productivities are $A_a = 0.18$, $A_b = 0.17$, and $A_c = 0.165$, i.e. financial intermediaries achieve a higher productivity with their investments than do households, and funds achieve a higher productivity than banks.

We run Monte-Carlo simulations of the models to get the real and financial moments of the model and the distribution of the state(s). We simulate 5 paths of 50,000 years each and discard the first 5,000 years as a burn-in phase.

3.2 EZ with two investors

The benchmark case is defined as the two-investor model with EZ preferences. We have $\gamma_a = 2.5$, $\gamma_c = 8.5$, and $\psi = 1.5$. We first look at the impact of the VaR-restriction on economic and financial variables.

Table 3a displays the average growth rates of capital μ_K in the first column. The annualized growth rate in the unrestricted model amounts to 1.78%. When there is a restriction, the growth rates first slightly increase to 1.92%, and then significantly decrease. The tightest restriction is given by $\bar{\sigma}_{FI} = 0.12$. In this case, the restriction binds in most states. The growth rate then drops to 1.03%.

The second column of Table 3a displays the percentage of states in which the restriction binds. As can be seen, this percentage is not a sufficient statistic to judge the real implications of the restriction. For $\bar{\sigma}_{FI} \leq 0.15$, the restriction binds (almost) 100% of the time. The three cases, however, differ in the amount of leverage that is still possible and that is larger for $\bar{\sigma}_{FI} = 0.15$ than for $\bar{\sigma}_{FI} = 0.12$. Since higher leverage allows intermediaries to better exploit their superior productivity, average growth rates are much larger for the less strict restriction, even though the percentage of binding states is comparable.

Table 4a gives the mean and standard deviation of further economic variables. It displays the average output $\bar{A}_t$, the investment rate ι_t, the price of capital p_t, the risk-free rate r_f, the excess return on capital, and the state variable x_t for different values of the restriction $\bar{\sigma}_{FI}$. The average productivity is 17.97% for the unrestricted case and the slightly restricted case and 17.45% for the most restricted case $\bar{\sigma}_{FI} = 0.12$. The differences in investment rates, shown in the next column, are more pronounced. Investment rates range from 12.71% in the unrestricted case to 12.82% in the slightly restricted case with $\bar{\sigma}_{FI} = 0.15$ and 11.91% in the most restricted case. This can also be seen from Equation (13), the pattern in investment rates follows the price of capital, which varies from 1.0814 in the unrestricted case to 1.0845 for $\bar{\sigma}_{FI} = 0.15$ and 1.0573 for the tightest restriction. Overall, the behavior of the growth rates described before is similar to the behavior of the investment rate and the price of capital.

The restriction also has an impact on the distribution of the state variable x, i.e., the wealth share of financial intermediaries. Starting from an initial value of $\bar{x} = 5\%$ at birth, the wealth share of intermediaries increases over time since they have higher productivity, hold riskier portfolios, and earn higher expected returns. On average, x is equal to 78.24% in the unrestricted case, 81.14% for $\bar{\sigma}_{FI} = 0.15$, and 63.48% in the most restricted case. Furthermore, the volatility of the state variable changes with the restriction. While the volatility is never large (around 6-7% in the unrestricted and loosely restricted cases), it drops to 0% for the tightest restriction. Hence, the composition of the economy remains (nearly) constant in this case. The reason for the decrease in volatility is that the volatility increases in the difference between the portfolio weights (see Equation (20)). As the restriction tightens, all portfolio weights are pushed closer to one, and this difference becomes smaller.

Figure 1 gives the density of the wealth share x of financial intermediaries. The black line represents the unrestricted case, and the brown line the most restricted case. When a restriction is imposed, the variance of x drops, confirming the results in Table 4a. This effect is much more pronounced for tight restrictions than for loose restrictions and the left tail is more reduced than the right tail. Furthermore, the distribution is shifted to the right for little to medium restriction levels, i.e., the average value of x increases. For the tightest restriction $\bar{\sigma}_{FI} = 0.12$, the mean and the volatility drop, confirming the values in Table 4a. As noted above, the capital growth rate first slightly increases with a tighter restriction. This can be explained by the decrease in variance. When the risk is lower, agents increase their investment rate and therefore capital grows faster, leading to an increase in the growth rate. When intermediaries are prohibited from taking leverage by the tightest restriction $\bar{\sigma}_{FI} = 0.12$, the expected average output is lower because it is a weighted average of the agents' productivities (see Equation (15)). Therefore, the expected return of capital is lower, decreasing the price and therefore the reinvestment rate ι. This leads to a decrease in the growth rate.

Finally, the restriction has an impact on asset pricing moments, as shown in Table 4a. The risk-free rate is around 1.39% in the unrestricted and slightly restricted cases, but drops sharply to -6.04% for the tightest restriction $\bar{\sigma}_{FI} = 0.12$. This decline is driven by two reinforcing channels. First, in the unrestricted economy, intermediaries lever up by borrowing at the risk-free rate to invest in capital, i.e. $\pi_{a,t} > 1$, which means they are net borrowers in the bond market. As the VaR constraint tightens, intermediaries are forced to reduce their leverage, which reduces their borrowing demand. Since the bond is in zero net supply, this creates excess lending in the bond market, and the risk-free rate must fall to restore equilibrium. Second, the restriction forces capital away from the more productive intermediaries toward the less productive households, lowering aggregate productivity $\bar{A}_t$ and thus expected consumption growth. Through the Euler equation, lower expected growth translates directly into a lower equilibrium risk-free rate. The equity risk premium displays a qualitatively similar but much more muted pattern as the risk-free rate. It is 7.76% in the unrestricted case, slightly drops when the restriction starts to bind, and falls to 7.10% in the most restricted case.

We now take a closer look at the mechanism. For this purpose, we analyze the dependence of variables on the state of the economy. In the economy with two investor types, the state variable is the wealth share of the financial intermediary. For $x = 0$, the economy is dominated by the more risk-averse households, while for $x = 1$, it is dominated by the less risk-averse financial intermediaries. Therefore, the average risk aversion in the economy decreases with x. The expected excess return on the risky stock inherits this monotonicity and is also a decreasing function of x as shown in Figure 2. When households dominate the economy ($x = 0$), the risk premium must be large enough to induce the more risk-averse households to hold all risky assets. When financial intermediaries dominate the economy ($x = 1$), the risk premium that induces the less risk-averse intermediaries to hold all risky assets is significantly lower.

Given the behavior of the risk premium, we can explain the portfolio weights shown in Figure 3. When one of the two investors accounts for the entire economy, their portfolio weight must be equal to 100% for financial markets to clear. Thus, the portfolio weight of the financial intermediary (type "a") is 100% for $x = 1$, and the portfolio weight of the household (type "c") is 100% for $x = 0$. The portfolio weight of the financial intermediary then increases when x goes from one to zero, and the financial intermediary takes on more and more leverage. The smaller the share of the financial intermediary in the economy, the greater its benefits from the high risk premium set by the more risk-averse household. On the other hand, the portfolio weights of the household drop from 100% for $x = 0$ to a level significantly below one for $x = 1$, since the risk premium, increasingly determined by the less risk-averse intermediaries, becomes less attractive to the more risk-averse households. It is noteworthy, that the household's portfolio weights are not monotonous, but slightly increase for high values of x. In contrast to that, the demand of the intermediaries is monotonous.

To explain the behavior of the household demand, we decompose the demand in myopic and hedging demand. The myopic demand is increasing in the Sharpe ratio. The hedging demand depends on the relative change in the value function when the state variable changes. Equation (23) gives the exact formulas. The results in the unrestricted case are given in Figure 4. The blue line shows the myopic demand, the red line the hedging demand, and the yellow line the total demand of each investor type. The intermediaries' demand (left plot) is monotonous except for a small decrease for $x \to 0$, which is consistent with Figure 3. What is more interesting is the composition of the households' demand. As noted before, the total demand is u-shaped. The household's myopic demand starts at one for $x = 0$, decreases sharply at first, and then declines approximately linearly for $x \geq 0.2$. The hedging demand, in contrast, exhibits an S-shape: it is zero at both boundaries of the state space, increases sharply for very small x, then decreases and turns negative around $x = 0.25$. It reaches its minimum around $x = 0.6$ and subsequently increases back toward zero, remaining negative throughout. Notably, the positive peak of the hedging demand is smaller in absolute value than its negative trough. For low values of x, the behavior of total demand is dominated by the myopic component. As x increases, however, the hedging demand becomes relatively more important, since its magnitude grows while the myopic demand declines roughly linearly. This interplay between the two components produces the U-shaped total demand observed in Figure 3. The shape of the hedging demand can be understood as follows. For very small values of x, the household benefits from a marginal increase in x, since the entry of more productive intermediaries improves overall economic conditions. The household therefore increases its exposure to the risky asset, generating positive hedging demand. As x grows further, however, the declining risk premium increasingly dominates: the risky asset co-moves positively with x and thus pays off precisely when the household's investment opportunities deteriorate, creating a negative hedging motive. For very high values of x, the output of capital is sufficiently large, as the more productive intermediaries own most of the capital, that the household seeks to participate in this prosperity and therefore increases its hedging demand again.

Returning to Figure 3: When a restriction is placed on the portfolio volatility of financial intermediaries, their high portfolio weights are capped. This is the case for a low wealth share x of financial intermediaries. Although they would like to profit from the attractive risk premium, largely set by households, the restriction prevents them from doing so. The tighter the restriction, the larger the value of x up to which the portfolio weights are capped and the lower the maximum level of leverage taken by the intermediaries.

The behavior of the portfolio weights can be used to explain the average productivity $\bar{A}$ in the economy shown in the left panel of Figure 5. It increases from $A_c = 0.165$ (when the households hold all wealth for $x = 0$) to $A_a = 0.18$ (when the financial intermediaries hold all wealth for $x = 1$). Since the financial intermediary, who has the higher productivity A_a, takes on more and more leverage as x decreases, the average productivity is a concave function of the wealth share x. A restriction limits the leverage of financial intermediaries and thus results in a smaller average productivity. With portfolio weights becoming nearly constant in x, average productivity becomes approximately linear in x in the constrained region.

Consistent with the behavior of productivity, the price of capital is also increasing in the wealth share of the financial intermediary (see Figure 6). Tighter restrictions result in a lower average productivity, which reduces the price of capital and thus also the investment rate. Note that the price of capital is always well above one, i.e., one unit of capital is always more expensive than one unit of consumption.

Figure 7 shows the volatility of the price of capital, which contributes to the return volatility. It is zero when one investor dominates the economy, and reaches its maximum at intermediate values of x. The maximum is at around $x = 0.3$ in the unrestricted case. When the restriction binds, the peak volatility is lower, and the value of x at which it begins to decline approximately coincides with the boundary between the constrained and unconstrained regions of the state space.

Finally, we look at consumption, which is depicted in Figure 8. The wealth-consumption ratio is a decreasing function of risk aversion, i.e., the more risk-averse an investor, the lower their wealth relative to consumption and the higher the consumption rate out of wealth. In line with this reasoning, consumption rates are larger for the more risk-averse households than for the less risk-averse financial intermediaries. Furthermore, good states induce investors to save more and to consume less, since with $\psi > 1$, the substitution effect dominates the income effect. In the unrestricted case, the consumption rate of the financial intermediary is low with around 0.5% for $x = 0$ (when it profits from the high expected excess return) and high with around 4.2% for $x = 1$ (when expected excess returns are smaller). For the same reason, the consumption rate is larger for the tightest restriction since limits on portfolio weights restrict investment in high-productivity projects, worsening economic prospects.

The right graph of Figure 5 shows the dividend yield, which equals the average consumption rate in the economy (see the market clearing equation (19)). It is equal to the (high) consumption rate of households for $x = 0$, and equal to the (low) consumption rate of financial intermediaries for $x = 1$. Overall, it is thus a decreasing function of x. The tighter the restriction, the worse the economic outlook, and the higher the consumption rate of the investors. This reasoning extends to the wealth-weighted average consumption rate.

To asses possible drawbacks of the constraint, we look at its impact on the agents' welfare through the certainty equivalent consumption (CE). It represents the amount of consumption per period that an agent has to get with certainty to have the same utility as the risky utility. We calculate it relative to the current wealth through

$$\frac{CE_i}{W_i} = G_i(x)^{\frac{1}{1-\psi_i}}, \tag{29}$$

the derivation is in the Appendix (chapter C.3). The results are given in Table 5. Analogously to the previous tables, the first column shows the level of the restriction. The second and third column show the certainty-equivalent consumption in this model specification, the rest are for other model specifications discussed later. The Column *type "a"* shows CE for intermediaries. It holds that $\frac{CE_a}{W_a} = 503$ in the unrestricted case. The certainty equivalent consumption increases with the restriction to 574.6161 for $\bar{\sigma}_{FI} = 0.135$, and then drops to 532.6531 for the tightest constraint. The values are on this level because of the exponent $\frac{1}{1-\psi_i}$ in Equation (29). The third column *type "c"* shows the certainty equivalent consumption for households. It holds that $\frac{CE_c}{W_c} = 232.1777$ in the unconstrained case. It then decreases to 194.1796 for $\bar{\sigma}_{FI} = 0.135$, and increases back to 205.641 for tightest constraint.

Two things are noteworthy in the behavior of the certainty equivalent consumption in the model: First, intermediaries have a higher certainty equivalent consumption than households and secondly, we see that the impact of the VaR-constraint on the certainty equivalent consumption is a hump-shaped function of the restriction for the intermediaries and a u-shaped one for the households. We start with analyzing the latter. The VaR constraint produces opposing non-monotonic patterns in certainty equivalent consumption: hump-shaped for intermediaries and U-shaped for households. A mild restriction primarily curbs the amplification mechanism by reducing intermediary leverage, which stabilizes the economy and raises intermediary CE. For households, however, this stability comes at a cost: fewer endogenous crises mean fewer episodes of wealth redistribution from intermediaries to households, lowering household CE. As the restriction tightens further, the dominant effect shifts from stabilization to misallocation. Intermediary CE declines as capital is increasingly forced toward less productive households, reducing aggregate productivity and expected consumption growth. Household CE, in contrast, recovers: capital market clearing requires them to absorb a larger share of the capital stock, the risk-free rate turns sharply negative — improving financing conditions — and the near-elimination of leverage volatility produces a stable consumption path that is particularly valuable given their high risk aversion.

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Across all restriction levels, intermediary certainty equivalent consumption exceeds that of households. This reflects two fundamental asymmetries in the model. First, intermediaries invest in more productive projects ($A_a > A_c$), generating higher expected consumption growth. Second, although both agents share the same intertemporal elasticity of substitution ψ, households are considerably more risk-averse ($\gamma_c > \gamma_a$). Since certainty equivalent consumption measures the riskless consumption level that yields the same utility as the actual stochastic consumption stream, a higher γ implies a larger discount for bearing uncertainty — even for an otherwise identical consumption process. Both channels work in the same direction, resulting in uniformly lower certainty equivalent consumption for households.

To summarize, restricting the portfolio volatility of financial intermediaries limits their ability to leverage their superior productivity, which reduces average productivity, lowers the price of capital and investment rates, and ultimately depresses economic growth. Additionally, the agents' welfare decreases, These effects are negligible for loose restrictions but become pronounced once the constraint binds tightly across most states. In the next section, we examine whether these findings are similar in an economy where all agents feature logarithmic preferences.

3.3 Log utility with two investors

We now look at the case of log utility. The assumption of log utility significantly eases calculations because the Hamilton-Jacobi-Bellman equation has a closed-form solution. This is why it is often used in the literature. Compared to our benchmark case, investors now have unit risk aversion and unit elasticity of intertemporal substitution, and their preferences no longer differ. Differences between their decisions are thus only driven by differences in their productivities, which are still present with $A_a > A_c$.

The growth rates of capital, output, and consumption (shown in Table 3b) are lower with log utility than with EZ preferences. They range from 1.14% in the unrestricted and mildly restricted cases to 0.87% under the tightest restriction. Furthermore, the restriction already binds in all states for $\bar{\sigma}_{FI} = 0.15$. The use of log utility for ease of calculation can thus lead to misleading conclusions. Reductions in the growth rate of capital due to VaR-restrictions on financial intermediaries are less than 0.3 percentage points and thus matter less for the economy. This is completely different from the EZ case, where growth rates of capital drop from 1.78% to 1.03%. Log utility thus leads to underestimation of the real implications of the restrictions on financial intermediaries.

We now turn to the economic quantities, shown in Table 4b. The average productivity in the unrestricted log case is equal to 17.07%. For the tightest restriction ($\bar{\sigma}_{FI} = 0.12$) it falls to 16.75%. The average investment rate is around 10.7% and thus lower than in the EZ case, reflecting the lower average price of capital of approximately 1.06. The risk-free rate is around 5%, and the equity risk premium is approximately 6%. Overall, the real and financial implications of the restriction are small. The intermediary wealth share represented by the state variable x is 22.6% in the unrestricted case and decreases to 16.59% for the tightest restriction. This is approximately 50 percentage points lower than in the EZ case. The volatility of x is 12.13% in the unrestricted case and decreases to 0.08% for the tightest restriction. Hence, in the unrestricted case, the volatility is almost twice as high for logarithmic preferences as for Epstein-Zin preferences. This can also be observed in Figure 9. The dispersion in the unrestricted and slightly restricted cases is substantially higher than for Epstein-Zin utility. The mean and volatility of x decrease with the tightness of the restriction.

The lower average wealth share of financial intermediaries is the result of several factors. First, with identical preferences, the leverage of financial intermediaries is due solely to their greater productivity, rather than their lower risk aversion. Consequently, the portfolio weight of financial intermediaries remains below 200% even in the unrestricted case (see Figure 10), while it easily reaches levels well above 250% in the unrestricted EZ case (see Figure 3). With smaller differences in portfolio weights between financial intermediaries and households, the difference in their expected portfolio returns becomes smaller as well. Second, consumption rates are equal to $\rho + \kappa = 4.5\%$ for both investor types, regardless of the state of the economy. Financial intermediaries thus no longer accumulate wealth faster through lower consumption rates, as they did under Epstein-Zin preferences, where the less risk-averse intermediaries consumed less and saved more.

3.4 Comparison of EZ and log utility

The preceding chapter has shown that restrictions on the portfolio volatility of financial intermediaries can have substantial real effects in an economy with two investor types. Under Epstein-Zin preferences, tight restrictions significantly reduce average productivity, investment rates, and economic growth by preventing intermediaries from leveraging their superior productivity. Under log utility, by contrast, these effects are economically negligible. The stark difference between the two cases demonstrates that the choice of preferences is not merely a technical convenience but has first-order implications for the assessment of financial regulation.

The shift from Epstein-Zin to log utility changes two preference parameters simultaneously: risk aversion increases to one and the elasticity of intertemporal substitution decreases to one. These two changes affect the economy through distinct channels. Lower risk aversion under EZ preferences drives the high leverage of intermediaries, while an elasticity of intertemporal substitution above one generates differential saving behavior between investor types. Under log utility, both channels are shut down at the same time, making it difficult to assess their individual contributions. In the following two chapters, we disentangle these effects by varying each parameter separately. We first examine the role of the elasticity of intertemporal substitution while keeping risk aversion at its benchmark level, and then analyze the role of risk aversion while maintaining unit elasticity of intertemporal substitution.

3.4.1 EZ with $\psi = 1$

When we switch from EZ preferences to log utility, we change both the (formerly different) risk aversion levels and the intertemporal elasticity of substitution to one. To better understand the mechanism, we examine several intermediate cases.

We start with the case in which we set $\psi = 1$[13] for both investors, but keep relative risk aversion levels at $\gamma_a = 2.5$ and $\gamma_c = 8.5$. The results are given in Table 6a. The growth rate in the unrestricted case increases from around 1.78% for $\psi = 1.5$ to 2.22% for $\psi = 1$. Under the tightest restriction, the growth rate drops to 0.87%. Hence, the growth rate in the unrestricted case is higher for $\psi = 1$ than for $\psi = 1.5$, but with the restriction, it is lower and on the same level as for logarithmic preferences.

To explain these findings, we look at the decisions of the investors with respect to portfolio weights and consumption rates. Consider first the portfolio weights. With the same difference in risk aversion levels as in the EZ case, intermediaries still want to take on significant leverage when x is small, as can be seen from a comparison of Figure 3 (for Epstein-Zin preferences with $\psi = 1.5$) with Figure 13 (for EZ preferences with $\psi = 1$). This is to be expected: under Epstein-Zin preferences, the portfolio allocation decision is primarily governed by the coefficient of relative risk aversion γ_i, not by the intertemporal elasticity of substitution ψ. Since we keep $\gamma_a = 2.5$ and $\gamma_c = 8.5$ unchanged, both agents perceive the same risk-return trade-off and choose nearly identical portfolio exposures as in the base case. The intermediary, being less risk-averse, still levers up aggressively at low values of x, while the more conservative household holds a smaller share of risky capital. As a consequence, the portfolio weight channel cannot explain the

[13] When solving the model numerically, we set $\psi = 1.001$ since $\psi = 1$ is a pole of the aggregator. Because our results for $\psi = 1.001$ and $\gamma_i = 1.001$ agree with the solution for the logarithmic case, we assume that this is equivalent to solving the model for an adjusted aggregator that satisfies $\psi = 1$.

differences in growth rates between $\psi = 1$ and $\psi = 1.5$. Under log utility, where the identical risk aversion levels significantly dampen leverage (see Figure 10), the growth rates are lower than under EZ, which can be explained by the .

We then examine the impact of preferences on the consumption rate. When $\psi = 1$, the income and substitution effects of such changes exactly offset each other: a higher expected return makes future consumption cheaper (substitution effect towards saving more), but also makes the agent wealthier (income effect towards consuming more). These two forces cancel perfectly, so that the consumption-wealth ratio becomes independent of the state of the economy.. With an intertemporal elasticity of substitution equal to one, consumption-wealth ratios are now constant and equal to $\rho + \kappa = 4.5\%$ (see Figure 14) as in the case of log utility. This is above the consumption rate of financial intermediaries in the benchmark EZ case and below that of households. The reason is that, in the base case with $\psi = 1.5 > 1$, the substitution effect dominates the income effect. Households, being significantly more risk-averse ($\gamma_c = 8.5$), perceive a low certainty-equivalent return on their wealth portfolio because they heavily discount risky future payoffs. Since their intertemporal elasticity of substitution exceeds one, they respond to this low risk-adjusted return by tilting toward current consumption—consuming more and saving less. Intermediaries, with their lower risk aversion ($\gamma_a = 2.5$), are less affected by this channel, so their consumption rate remains comparatively low. When we set $\psi = 1$, this mechanism is shut down entirely: both agents maintain a fixed consumption-wealth ratio regardless of risk-adjusted returns, and the elevated consumption rate of households in the base case drops to the common level of 4.5%. Hence, the dividend yield, which equals the aggregate consumption rate in our model, is constant at 0.045 and thus significantly lower than in the base case where it ranges between 0.075 and 0.045.

With lower aggregate consumption rates, the investment rate in the unrestricted case increases to 12.77% compared to 12.02% in the base case (see Table 7a and Table 4a). A similar pattern can be seen in the price of capital, which increases to 1.0830 from 1.0814. The higher investment rate, in turn, leads to a higher growth rate.

In summary, the shift from $\psi = 1.5$ to $\psi = 1$ raises the growth rate not because agents become more willing to substitute consumption intertemporally, but precisely because they become *less* responsive to the investment opportunity set. At $\psi = 1.5$, households react to their low risk-adjusted returns by consuming a large share of their wealth. At $\psi = 1$, this response vanishes, freeing up resources for investment and capital accumulation.

While this explains why growth is higher at $\psi = 1$ than at $\psi = 1.5$, the unrestricted growth rate of 2.22% at $\psi = 1$ remains well above the log utility benchmark. Since consumption-wealth ratios are already constant at 4.5% under both $\psi = 1$ and log utility, the consumption-savings margin cannot account for this remaining gap. Instead, it is driven entirely by the portfolio allocation channel. With heterogeneous risk aversion levels of $\gamma_a = 2.5$ and $\gamma_c = 8.5$, the less risk-averse intermediary takes on substantial leverage, concentrating risk-bearing in the hands of the agent better suited to bear it. This raises the price of capital and stimulates investment beyond the level that prevails under log utility. The interpretation is confirmed by the behavior of the restricted economy: once leverage constraints bind, the intermediary can no longer amplify its exposure, and the growth rate drops to 0.87% which is essentially the same level as under log preferences. Under log utility, both agents share a risk aversion of one, eliminating any heterogeneity that could be exploited through leverage. There are no gains from specialization in risk-bearing, no role for intermediation, and the growth-enhancing effect of leverage disappears entirely.

Having isolated the role of the elasticity of intertemporal substitution, we now examine the complementary case in which both investors share the same risk aversion and vary ψ between 1 and 1.5.

3.4.2 EZ with identical γ

The next intermediate case we consider is one in which investors share the same level of relative risk aversion, but this common value differs from one. Specifically, we lower households' risk aversion so that both agents have $\gamma = 2.5$ to disentangle the effect of differences in risk-aversion from unit risk-aversion. Additionally, we vary the value of the intertemporal elasticity of substitution ψ between 1 and 1.5. The growth rates that result from the Monte-Carlo simulations are provided in Tables 6b and 6c, while the corresponding economic moments appear in Tables 7b and 7c .

We first examine the case with an intertemporal elasticity of substitution of $\psi = 1.5$, consistent with the baseline calibration. In this configuration, the aggregate growth rate is 1.78% in the unrestricted case and drops to 1.58% under the tightest constraint. Hence, equalizing risk aversion across agents reduces the sensitivity of growth to the intermediary restriction relative to the benchmark model.

The economic quantities reflect the same pattern. Setting $\gamma_c = \gamma_a = 2.5$ alters the risk–return trade-offs in the economy and mitigates the effect of the constraint. The average productivity $\bar{A}$ now lies between 16.97% and 17.22%, compared with 17.97% in the base case. The investment rate varies between 12.46% and 12.70%, slightly lower than the benchmark value of 12.71%. The price of capital is also lower than in the base case (between 1.0737 and 1.0810 compared to 1.0814).

By contrast, the risk-free rate rises substantially: from 1.39% in the base case to values between 4.68% and 4.87%. The excess return slightly decreases from 7.76% to values between 6.58% and 7.00%. Hence, even though the growth rates are fairly similar in the unrestricted case compared to the base model, the impact of equalizing risk aversion on asset pricing moments is more pronounced. We examine the mechanisms behind these shifts next, starting with the portfolio allocation channel.

With equal risk aversion, both agents perceive the same risk–return trade-off and desire similar portfolio exposures. Intermediaries no longer take on substantial leverage; their maximum exposure is roughly 140%, compared to over 250% in the base case (compare Figures 3 and 18). Households, now less risk-averse, demand a higher share of risky capital. As a result, risk-sharing in the economy becomes more equal, with the remaining differences in portfolio weights stemming solely from differences in productivity. Since leverage constraints are far less binding in this configuration, they cause less distortion when imposed—which explains why the sensitivity of the growth rate to the restriction is markedly lower than in the benchmark.

The more equal risk-sharing is also reflected in asset prices. Figure 22 shows that the volatility of the price of capital, $\sigma_{p,t}$, is about 10% of the volatility in the base case (see Figure 7). With less volatile capital prices, the risk premium declines—consistent with the lower average risk aversion in the economy. At the same time, agents' demand for the risk-free asset falls, since there is less need to hedge against price fluctuations. The risk-free rate must therefore rise to clear the bond market.

While these shifts in risk-sharing and asset prices are substantial, the unrestricted growth rate of 1.78% coincides with the benchmark. This reflects two approximately offsetting forces. On the one hand, the economy loses the gains from heterogeneous risk-bearing: without differences in risk aversion, there is no scope for intermediaries to improve aggregate risk allocation through leverage. Moreover, a larger share of capital is now held by the less productive household sector, which lowers average productivity $\bar{A}$. On the other hand, the lower aggregate risk aversion raises the overall demand for capital, which supports the price of capital and thus the investment rate despite the decline in $\bar{A}$ (compare Figures 21 and 6). These opposing effects roughly cancel, leaving the unrestricted growth rate unchanged.

In summary, equalizing the agents' risk aversion has a minor impact on the unrestricted growth rate but substantially reduces the economy's sensitivity to leverage constraints. With homogeneous risk preferences, intermediaries have little reason to lever up, so that restricting leverage causes only modest distortions.

In a second step, we not only set the risk aversion levels to $\gamma_c = \gamma_a = 2.5$, but also lower the elasticity of intertemporal substitution to $\psi = 1$. The only remaining difference to logarithmic preferences is that both agents' risk aversion exceeds one.

The unrestricted growth rate falls to 1.20%, compared to 1.78% for $\psi = 1.5$ with the same risk aversion levels. Under the tightest restriction $\bar{\sigma}_{FI} = 0.12$, the growth rate is 0.87%—the same value as in the other two configurations with $\psi = 1$ (see Tables 3b and 6a).

The economic quantities confirm this pattern. Average productivity, the investment rate ι, and the price of capital are on a similar level as in the logarithmic case (see Tables 7c and 4b). The risk-free rate, however, is approximately 2.6%, roughly half of its value under log preferences. The higher risk aversion ($\gamma = 2.5$ versus $\gamma = 1$) increases agents' demand for the risk-free asset, pushing its return down relative to the log case.

To understand these results, we examine the two channels identified above in turn. As expected, the portfolio weights do not react to the change in the intertemporal elasticity of substitution, which can be seen from a comparison of Figures 23 (for $\psi = 1$) and 18 (for $\psi = 1.5$). This confirms the separation property of Epstein-Zin preferences: portfolio allocations are governed by risk aversion, not by the EIS.

The consumption channel, however, operates in the opposite direction compared to the benchmark case with heterogeneous risk aversion. Recall that in the benchmark, setting $\psi = 1$ *raised* the growth rate: the highly risk-averse households ($\gamma_c = 8.5$) had perceived a low certainty-equivalent return and, at $\psi = 1.5$, had responded by consuming a large share of their wealth. Setting $\psi = 1$ eliminated this response and reduced their consumption rate to the constant level of $\rho + \kappa = 4.5\%$, freeing up resources for investment.

With homogeneous risk aversion of $\gamma = 2.5$, the mechanism reverses. Both agents now face a relatively high certainty-equivalent return, and at $\psi = 1.5$ they respond to this favorable investment environment by saving more, i.e., the substitution effect dominates. When we set $\psi = 1$, this channel is shut down: consumption-wealth ratios become constant at 4.5% regardless of the investment opportunity set (see Figure 24). Since agents can no longer respond to the high risk-adjusted return by saving more, aggregate consumption rises relative to the $\psi = 1.5$ case, investment falls, and the growth rate declines from 1.78% to 1.20%.

Under the tightest restriction, the growth rate of 0.87% coincides with the values obtained in the other $\psi = 1$ configurations. This convergence is intuitive: with $\psi = 1$, the consumption rate is fixed at 4.5% regardless of the specification. Once leverage constraints also eliminate differences in portfolio allocations, the remaining variation comes only from differences in average productivity. As the constraint tightens, intermediaries hold less capital, lowering $\bar{A}$ (from 16.47% to 15.0%, see Table 7c) and thus the price of capital and the investment rate, until the growth rate settles at the same level as in the log-utility economy.

Taken together, the intermediate cases reveal that the growth dynamics of our model are shaped by the interaction of three distinct forces: (i) the *level* of risk aversion, which determines how much agents discount risky returns; (ii) *heterogeneity* in risk aversion, which creates gains from leverage and intermediation; and (iii) the intertemporal elasticity of substitution, which governs whether agents translate changes in risk-adjusted returns into changes in their consumption-savings decision. Only when all three channels are active—as in the benchmark calibration—do leverage constraints generate large effects on economic growth.

3.5 EZ with three investors

So far, we have looked at a model that consists of intermediaries and households. While this helps to better understand model dynamics and is computationally simpler, it misses heterogeneity in the financial sector. Kargar (2021) shows that this heterogeneity is important in explaining asset prices. For this reason, we now split the intermediary sector into funds (type "a") and banks (type "b"). This setup of agents is similar to Kargar (2021). While he assumed an endowment economy, we now want to explore the real effects of heterogeneity in the intermediary sector.

The newly introduced type "b" investor has a relative risk aversion of 5.5 and a productivity of 17%, which places him between type "a" and type "c" (still households) in terms of both his risk aversion and his productivity. See sections 2.6 and D.1 for details.

Table 3c shows that the growth rate is affected by the introduction of the third investor. The growth rate of capital amounts to 2.08% in the unrestricted case and 0.89% in the case of the tightest restriction $\bar{\sigma}_{FI} = 0.12$. The loss due to the tightest restriction amounts to more than 1 percentage point, which is sslightly larger than in the two-investor case. While the absolute levels thus differ, the real implications of the restriction are the same. This implies, in particular, that the use of three instead of two investors does not bias the overall conclusion.

Table 4c gives mean and variance of several additional economic quantities. Looking at the average productivity $\bar{A}$, we see similar values for the unrestricted and the slightly restricted case ($\bar{A} \approx 17.92\%$ in both the two- and three-investor setting). Only for the tightest restriction, $\bar{\sigma}_{FI} = 0.12$, the drop in average productivity is larger for the three-investor economy (17.37% versus 17.45%). The behavior of the investment rate ι is similar to the behavior of the growth rate. In the unrestricted case, the investment rate in the three investor case is higher than for two investors (12.99% compared to 12.71%). For the tightest restriction, this relation switches. At a restriction level of $\bar{\sigma}_{FI} = 0.12$, the investment rate is 11.81% compared to 11.91% in the two-investor configuration with the same level of restriction. A similar behavior can be seen with the price of capital. The risk-free rate is higher in the three-investor case, amounting to 2.74% compared to 1.39% in the two-investor benchmark. Moreover, the decline in the risk-free rate under tight restrictions is less pronounced in the three-investor configuration, with a drop to only -3.22% versus a more substantial decrease to -6.04% in the two-investor case. The inverse pattern can be seen in the excess return: In the three-investor setting, the excess return is always 6-9 basis points below the two-investor setting.

We again look at the mechanisms that lead to these results. The wealth share of agent type "a" is given by $xy = 72.01\%$[14] for the unrestricted case, which indicates a slightly lower wealth share for the agent with the highest productivity compared to the two-investor model, where $x = 78.24\%$. Households own on average 27.96% of the economy (compared to 21.76% in the two investor model without a restriction), while type "b" owns about 1.53% of wealth. With the tightest restriction $\bar{\sigma}_{FI} = 0.12$, the share of funds drops to 47.36%[15]. Additionally, the share of the households drops to 26.73%. Banks are the only agents that increase their share to 25.91% [16]. In this case, the restriction is so tight that funds can no longer take any leverage (see Figure 26). Instead, banks (having a lower risk-aversion than households) absorb the capital and invest approximately 200% of their wealth into capital. Because banks have a higher productivity than households, which would otherwise have to take over, the average productivity decreases less with a tighter restriction. This also impacts the other economic variables.

An additional explanation for the behavior of the growth rate in the three-investor economy is related to changes in aggregate consumption and investment demand. Relative to the two-investor benchmark, the introduction of banks (type "b") with intermediate risk aversion and productivity affects the distribution of consumption across agents. In the restricted economy, banks take over part of the risky investment from funds, but consume more than funds would have done in the absence of constraints. As a result, aggregate consumption is higher in the three-investor economy than in the two-investor case

[14] $0.7051 \cdot 0.9792$ with x and y from Table 4c
[15] $0.7327 \cdot 0.6464$ with x and y from Table 4c
[16] $0.7327 \cdot (1 - 0.6464)$ with x and y from Table 4c

under tight restrictions. This effect is enforced mechanically through the goods market clearing condition (equations (52) and (19)). For a given level of output, a higher consumption share implies that fewer resources are available for investment, which requires a lower equilibrium price of capital. The lower price of capital, in turn, reduces the optimal investment rate through equation (13), thereby lowering capital accumulation and economic growth.

Overall, introducing heterogeneity in the intermediary sector enriches the model dynamics but preserves, and in fact sharpens, the core mechanism identified in the two-investor economy.

In the unrestricted case, the presence of banks raises the growth rate from 1.78% to 2.08%. This is because an additional layer of intermediation deepens the scope for risk-sharing: funds can offload some risk to banks rather than directly to the least productive households, keeping more capital in the hands of highly productive agents.

Under tight leverage constraints, however, banks play a more ambiguous role. On the one hand, they act as a *buffer*: when funds can no longer lever up, banks (being less risk-averse than households) absorb part of the risky capital that would otherwise migrate to the household sector. Since banks are more productive than households, this partial substitution limits the decline in average productivity $\bar{A}$. On the other hand, banks also *consume more* than the highly leveraged funds they replace. The resulting increase in aggregate consumption crowds out investment through the goods market clearing condition, lowering the equilibrium price of capital and the investment rate. This consumption leakage explains why, under the tightest restriction, the three-investor economy actually produces a slightly lower growth rate (0.89%) and investment rate (11.81%) than the two-investor benchmark (0.91% and 11.91%, respectively).

The net effect of adding banks is therefore twofold: they amplify the unrestricted growth rate by improving risk allocation, but they also amplify the growth *loss* from leverage constraints; the gap between the unrestricted and restricted growth rate widens from approximately 0.9 to 1.2 percentage points. This confirms that our main conclusions are robust to a richer intermediary structure. If anything, the real costs of financial restrictions are larger when the financial sector itself is heterogeneous, because constraints not only prevent efficient risk-bearing but also trigger a reallocation toward agents whose higher consumption further depresses capital accumulation.

3.6 Log utility with three investors

The results for log utility with three investors are very similar to the two investor case. Again, the growth rates are smaller with log utility than with EZ preferences, see Table 3d. The use of log utility, which eases computations, thus again results in a severe underestimation of the real implications of restrictions.

4 Conclusion

This paper analyzes the role of portfolio restrictions on financial intermediaries in a continuous-time production economy with heterogeneous agents and Epstein–Zin preferences. Our model incorporates financial intermediaries and households, with intermediaries generally being less risk-averse and more productive. We introduce a Value-at-Risk constraint that limits the risk-taking of these intermediaries. We demonstrate that this regulatory friction substantially impacts investment, depresses capital prices, and significantly alters the growth rate of aggregate consumption, often exhibiting an inversely U-shaped response to the constraint's tightness.

Our comparison of preference specifications highlights that log-utility economies, frequently used for tractability, substantially underestimate these effects and can lead to misleading conclusions about the relevance of regulation. Extending the model to a heterogeneous intermediary sector confirms that the main mechanism remains intact: restrictions on the most productive intermediaries depress growth even when other intermediaries can absorb part of the risk.

This work bridges the intermediary asset pricing literature with models of real economic activity, offering insights for both theoretical modeling and policy design. It providse new insights into the macroeconomic consequences of financial regulation and suggests that policy measures constraining intermediaries' risk-taking should carefully weigh the trade-off between financial stability and long-run growth.

References

Adrian, Tobias, Erkko Etula, and Tyler Muir, 2014, Financial Intermediaries and the Cross-Section of Asset Returns, *Journal of Finance* 69, 2557–2596.

Baron, Matthew, and Tyler Muir, 2022, Intermediaries and asset prices: International evidence since 1870, *Review of Financial Studies* 35, 2144–2189.

Bauer, Michael D., Ben S. Bernanke, and Eric Milstein, 2023, Risk Appetite and the Risk-Taking Channel of Monetary Policy, *Journal of Economic Perspectives* 37, 77–100.

Bhamra, Harjoat S., and Raman Uppal, 2013, Asset Prices with Heterogeneity in Preferences and Beliefs, *Review of Financial Studies* 27, 519–580.

Brunnermeier, Markus K., and Yuliy Sannikov, 2014, A Macroeconomic Model with a Financial Sector, *American Economic Review* 104, 379–421.

Campbell, John Y., 2018, *Financial decisions and markets: A course in asset pricing.* (Princeton University Press).

Cochrane, John H., 1991, Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations, *The Journal of Finance* 46, 209–237.

Cochrane, John H, 1996, A Cross-Sectional Test of an Investment-Based Asset Pricing Model, *Journal of Political Economy* 104, 572–621.

Du, Wenxin, Benjamin Hébert, and Amy Wang Huber, 2023, Are Intermediary Constraints Priced?, *The Review of Financial Studies* 36, 1464–1507.

Dumas, Bernard, 1989, Two-Person Dynamic Equilibrium in the Capital Market, *Review of Financial Studies* 2, 157–188.

Elenev, Vadim, Tim Landvoigt, and Stijn Van Nieuwerburgh, 2021, A Macroeconomic Model With Financially Constrained Producers and Intermediaries, *Econometrica* 89, 1361–1418.

Gârleanu, Nicolae, and Stavros Panageas, 2015, Young, Old, Conservative, and Bold: The Implications of Heterogeneity and Finite Lives for Asset Pricing, *Journal of Political Economy* 123, 670–685.

Gertler, Mark, and Nobuhiro Kiyotaki, 2015, Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy, *American Economic Review* 105, 2011–43.

Gârleanu, Nicolae, and Lasse Pedersen, 2011, Margin-Based Asset Pricing and Deviations from the Law of One Price, *Review of Financial Studies* 24, 1980–2022.

Haddad, Valentin, and Tyler Muir, 2021, Do intermediaries matter for aggregate asset prices?, *Journal of Finance* 76, 2719–2761.

He, Zhiguo, Bryan Kelly, and Asaf Manela, 2017, Intermediary asset pricing: New evidence from many asset classes, *Journal of Financial Economics* 126, 1–35.

He, Zhiguo, and Arvind Krishnamurthy, 2012, A Model of Capital and Crises, *Review of Economic Studies* 79, 735–777.

He, Zhiguo, and Arvind Krishnamurthy, 2013, Intermediary asset pricing, *American Economic Review* 103, 732–770.

He, Zhiguo, and Arvind Krishnamurthy, 2019, A Macroeconomic Framework for Quantifying Systemic Risk, *American Economic Journal: Macroeconomics* 11, 1–37.

Kargar, Mahyar, 2021, Heterogeneous intermediary asset pricing, *Journal of Financial Economics* 141, 505–532.

Krishnamurthy, Arvind, and Wenhao Li, 2024, Dissecting Mechanisms of Financial Crises: Intermediation and Sentiment, *Journal of Political Economy* p. null.

Maxted, Peter, 2023, A Macro-Finance Model with Sentiment, *The Review of Economic Studies* 91, 438–475.

Wang, Jiang, 1996, The term structure of interest rates in a pure exchange economy with heterogeneous investors, *Journal of Financial Economics* 41, 75–110.

A Tables

	Preferences	
$\psi_a = \psi_b = \psi_c$	EIS of all agents	1.5
γ_a	Risk Aversion of a (funds)	2.5
γ_b	Risk Aversion of b (banks)	5.5
γ_c	Risk Aversion of c (households)	8.5
ρ	Rate of time preference	0.03
	Production and demography	
δ	Depreciation rate	0.10
σ_K	volatility of capital	0.12
χ	investment adjustment costs	3
κ	mortality/birth rate	0.017
$\bar{x}$		0.05
$\bar{y}$		0.20
A_a	productivity of capital held by agent a	0.18
A_b	productivity of capital held by agent b	0.17
A_c	productivity of capital held by agent c	0.165

Table 2: Parameterization of the model
The table gives the parameters in the model with three investors. In the model with two investors, only agents of type a (now financial intermediaries) and type c (still households) are present.

(a) Two investors with Epstein-Zin preferences

Restriction	μ_K (%)	μ_C (%)	μ_Y (%)	restricted
unres	1.78	1.78	1.78	0.00
0.3	1.79	1.79	1.79	0.01
0.2	1.87	1.87	1.87	0.21
0.15	1.92	1.92	1.92	0.99
0.135	1.73	1.73	1.73	1.00
0.12	1.03	1.03	1.03	1.00

(b) Two investors with logarithmic preferences

Restriction	μ_K (%)	μ_C (%)	μ_Y (%)	restricted
unres	1.14	1.14	1.14	0.00
0.3	1.14	1.14	1.14	0.00
0.2	1.12	1.12	1.12	0.82
0.15	1.00	1.00	1.00	1.00
0.135	0.93	0.93	0.93	1.00
0.12	0.87	0.87	0.87	1.00

(c) Three investors with Epstein-Zin preferences

Restriction	μ_K (%)	μ_C (%)	μ_Y (%)	restricted
unres	2.08	2.08	2.08	0.00
0.3	2.09	2.09	2.09	0.00
0.2	2.13	2.13	2.13	0.08
0.15	2.06	2.06	2.06	0.76
0.135	1.84	1.84	1.84	1.00
0.12	0.89	0.89	0.89	1.00

(d) Three investors with logarithmic preferences

Restriction	μ_K (%)	μ_C (%)	μ_Y (%)	restricted
unres	0.97	0.97	0.97	0.00
0.3	0.97	0.97	0.97	0.00
0.2	0.95	0.95	0.95	0.94
0.15	0.82	0.82	0.82	1.00
0.135	0.78	0.78	0.78	1.00
0.12	0.75	0.75	0.75	1.00

Table 3: Growth in the economy

The table gives the growth rate of capital K_t (μ_K). The column *restricted* shows the probability that the restriction is binding.

(a) Two investors with Epstein-Zin preferences

restr	$\bar{A}$	ι	Price	r_f	$E[r]-r_f$	x	y
unres	0.1797	0.1271	1.0814	0.0139	0.0776	0.7824	
	(0.0002)	(0.0018)	(0.0053)	(0.0098)	(0.0015)	(0.0662)	
0.3	0.1797	0.1272	1.0817	0.0140	0.0776	0.7836	
	(0.0002)	(0.0017)	(0.0051)	(0.0100)	(0.0015)	(0.0652)	
0.2	0.1797	0.1280	1.0840	0.0124	0.0774	0.7929	
	(0.0004)	(0.0016)	(0.0048)	(0.0142)	(0.0011)	(0.0577)	
0.15	0.1787	0.1282	1.0845	-0.0135	0.0753	0.8114	
	(0.0006)	(0.0013)	(0.0040)	(0.0115)	(0.0001)	(0.0348)	
0.135	0.1776	0.1262	1.0785	-0.0415	0.0736	0.7836	
	(0.0003)	(0.0008)	(0.0024)	(0.0035)	(0.0001)	(0.0210)	
0.12	0.1745	0.1191	1.0573	-0.0604	0.0710	0.6348	
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	

(b) Two investors with logarithmic preferences

restr	$\bar{A}$	ι	Price	r_f	$E[r]-r_f$	x	y
unres	0.1707	0.1202	1.0607	0.0506	0.0674	0.2260	
	(0.0026)	(0.0022)	(0.0065)	(0.0013)	(0.0021)	(0.1213)	
0.3	0.1707	0.1202	1.0607	0.0506	0.0674	0.2260	
	(0.0026)	(0.0022)	(0.0065)	(0.0013)	(0.0021)	(0.1213)	
0.2	0.1705	0.1200	1.0601	0.0503	0.0672	0.2250	
	(0.0025)	(0.0021)	(0.0062)	(0.0014)	(0.0020)	(0.1112)	
0.15	0.1690	0.1188	1.0564	0.0486	0.0658	0.2135	
	(0.0009)	(0.0007)	(0.0022)	(0.0002)	(0.0007)	(0.0476)	
0.135	0.1682	0.1182	1.0546	0.0481	0.0652	0.1926	
	(0.0004)	(0.0003)	(0.0009)	(0.0000)	(0.0003)	(0.0222)	
0.12	0.1675	0.1176	1.0527	0.0478	0.0646	0.1659	
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0008)	

(c) Three investors with Epstein-Zin preferences

restr	$\bar{A}$	ι	Price	r_f	$E[r]-r_f$	x	y
unres	0.1792	0.1299	1.0896	0.0274	0.0737	0.7204	0.9792
	(0.0003)	(0.0015)	(0.0044)	(0.0042)	(0.0004)	(0.0773)	(0.0085)
0.3	0.1792	0.1300	1.0900	0.0275	0.0737	0.7227	0.9794
	(0.0003)	(0.0014)	(0.0043)	(0.0044)	(0.0004)	(0.0762)	(0.0079)
0.2	0.1794	0.1305	1.0914	0.0269	0.0738	0.7386	0.9806
	(0.0004)	(0.0015)	(0.0044)	(0.0075)	(0.0005)	(0.0692)	(0.0066)
0.15	0.1790	0.1300	1.0899	0.0122	0.0734	0.7793	0.9821
	(0.0007)	(0.0017)	(0.0050)	(0.0143)	(0.0001)	(0.0453)	(0.0041)
0.135	0.1779	0.1278	1.0834	-0.0153	0.0725	0.7871	0.9747
	(0.0005)	(0.0012)	(0.0035)	(0.0071)	(0.0001)	(0.0304)	(0.0036)
0.12	0.1737	0.1181	1.0542	-0.0322	0.0701	0.7327	0.6464
	(0.0001)	(0.0003)	(0.0010)	(0.0024)	(0.0000)	(0.0253)	(0.0300)

(d) Three investors with logarithmic preferences

restr	$\bar{A}$	ι	Price	r_f	$E[r] - r_f$	x	y
unres	0.1688	0.1185	1.0556	0.0497	0.0657	0.1780	0.6689
	(0.0024)	(0.0020)	(0.0060)	(0.0011)	(0.0020)	(0.1043)	(0.1675)
0.3	0.1688	0.1186	1.0559	0.0496	0.0656	0.1779	0.6687
	(0.0024)	(0.0020)	(0.0061)	(0.0011)	(0.0020)	(0.1043)	(0.1675)
0.2	0.1685	0.1184	1.0552	0.0494	0.0654	0.1742	0.6836
	(0.0021)	(0.0018)	(0.0053)	(0.0010)	(0.0017)	(0.0910)	(0.1298)
0.15	0.1670	0.1171	1.0513	0.0484	0.0641	0.1382	0.6270
	(0.0006)	(0.0005)	(0.0014)	(0.0001)	(0.0005)	(0.0358)	(0.0506)
0.135	0.1664	0.1167	1.0500	0.0483	0.0637	0.1169	0.5501
	(0.0002)	(0.0002)	(0.0006)	(0.0000)	(0.0002)	(0.0205)	(0.0308)
0.12	0.1660	0.1163	1.0490	0.0482	0.0633	0.1003	0.4606
	(0.0001)	(0.0001)	(0.0002)	(0.0001)	(0.0001)	(0.0126)	(0.0560)

Table 4: Economic variables in the simulation

The table gives the mean and the standard deviation (in brackets) of some central asset pricing moments and real values. The parameters for the model are given in Table 2. The *preference* column denotes whether the agents have logarithmic or recursive preferences. The *restr* column gives the value of $\bar{\sigma}_{FI}$ or denotes that there is no restriction in place (*unres*).

	two investors		two investors with equal γ		three investors		
restriction	type "a"	type "c"	type "a"	type "c"	type "a"	type "b"	type "c"
unres	503.003	232.1447	698.8054	533.6913	611.0388	337.4421	291.8353
0.3	505.2457	231.6929	698.8054	533.6913	613.5889	337.5632	290.9647
0.2	528.1448	225.6049	698.8054	533.6913	630.1472	331.8449	279.4489
0.15	574.8411	200.5356	696.2305	533.6227	632.3143	312.2974	240.8664
0.135	574.6161	194.1796	687.2883	534.0972	603.5174	312.7654	218.0667
0.12	532.6561	205.641	673.3608	534.7848	532.7679	327.1669	217.149

Table 5: Certainty Equivalents

A.1 Intermediate Cases with two investors

(a) $\psi_i = 1$

Restriction	μ_K (%)	μ_C (%)	μ_Y (%)	restricted
unres	1.81	1.81	1.81	0.00
0.3	1.81	1.81	1.81	0.02
0.2	1.81	1.81	1.81	0.46
0.15	1.65	1.65	1.65	1.00
0.135	1.37	1.37	1.37	1.00
0.12	0.87	0.87	0.87	1.00

(b) Identical $\gamma = 2.5$

Restriction	μ_K (%)	μ_C (%)	μ_Y (%)	restricted
unres	1.78	1.78	1.78	0.00
0.3	1.78	1.78	1.78	0.00
0.2	1.79	1.79	1.79	0.69
0.15	1.75	1.75	1.75	1.00
0.135	1.68	1.68	1.68	1.00
0.12	1.58	1.58	1.58	1.00

(c) Identical $\gamma = 2.5$ and $\psi = 1$

Restriction	μ_K (%)	μ_C (%)	μ_Y (%)	restricted
unres	1.20	1.20	1.20	0.00
0.3	1.20	1.20	1.20	0.00
0.2	1.20	1.20	1.20	0.00
0.15	1.14	1.14	1.14	1.00
0.135	1.00	1.00	1.00	1.00
0.12	0.87	0.87	0.87	1.00

Table 6: Growth in the Economy in the intermediate cases

The table gives the growth rate of capital K_t (μ_K). The column *restricted* shows the probability that the restriction is binding.

(a) $\psi_i = 1$

γ_a	γ_c	ψ	restr	$\bar{A}$	ι	Price	r_f	$E[r] - r_f$	x
2.5	8.5	1	unres	0.1797	0.1277	1.0830	0.0072	0.0781	0.7495
				(0.0005)	(0.0004)	(0.0013)	(0.0133)	(0.0009)	(0.0875)
2.5	8.5	1	0.3	0.1798	0.1277	1.0832	0.0073	0.0781	0.7510
				(0.0005)	(0.0004)	(0.0013)	(0.0137)	(0.0009)	(0.0871)
2.5	8.5	1	0.2	0.1800	0.1276	1.0827	0.0011	0.0780	0.7544
				(0.0016)	(0.0008)	(0.0023)	(0.0191)	(0.0013)	(0.0748)
2.5	8.5	1	0.15	0.1771	0.1255	1.0765	-0.0376	0.0739	0.7059
				(0.0008)	(0.0007)	(0.0020)	(0.0086)	(0.0007)	(0.0495)
2.5	8.5	1	0.135	0.1735	0.1226	1.0677	-0.0584	0.0702	0.5267
				(0.0005)	(0.0004)	(0.0012)	(0.0007)	(0.0004)	(0.0310)
2.5	8.5	1	0.12	0.1675	0.1175	1.0526	-0.0602	0.0646	0.1670
				(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0008)

(b) Identical $\gamma = 2.5$

γ_a	γ_c	ψ	restr	$\bar{A}$	ι	Price	r_f	$E[r] - r_f$	x
2.5	2.5	1.5	unres	0.1722	0.1270	1.0810	0.0487	0.0700	0.3330
				(0.0021)	(0.0025)	(0.0074)	(0.0009)	(0.0019)	(0.1208)
2.5	2.5	1.5	0.3	0.1722	0.1270	1.0810	0.0487	0.0700	0.3330
				(0.0021)	(0.0025)	(0.0074)	(0.0009)	(0.0019)	(0.1208)
2.5	2.5	1.5	0.2	0.1722	0.1270	1.0811	0.0486	0.0699	0.3376
				(0.0021)	(0.0024)	(0.0071)	(0.0010)	(0.0019)	(0.1128)
2.5	2.5	1.5	0.15	0.1714	0.1264	1.0793	0.0473	0.0680	0.3622
				(0.0009)	(0.0010)	(0.0030)	(0.0001)	(0.0007)	(0.0505)
2.5	2.5	1.5	0.135	0.1707	0.1257	1.0770	0.0470	0.0670	0.3475
				(0.0004)	(0.0005)	(0.0015)	(0.0000)	(0.0003)	(0.0257)
2.5	2.5	1.5	0.12	0.1697	0.1246	1.0737	0.0468	0.0658	0.3164
				(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0000)	(0.0014)

(c) Identical $\gamma = 2.5$ and $\psi_i = 1$

γ_a	γ_c	ψ	restr	$\bar{A}$	ι	Price	r_f	$E[r] - r_f$	x
2.5	2.5	1	unres	0.1716	0.1210	1.0629	0.0263	0.0691	0.3611
				(0.0010)	(0.0008)	(0.0025)	(0.0004)	(0.0009)	(0.0615)
2.5	2.5	1	0.3	0.1716	0.1210	1.0629	0.0263	0.0691	0.3611
				(0.0010)	(0.0008)	(0.0025)	(0.0004)	(0.0009)	(0.0615)
2.5	2.5	1	0.2	0.1716	0.1210	1.0629	0.0263	0.0691	0.3611
				(0.0010)	(0.0008)	(0.0025)	(0.0004)	(0.0009)	(0.0615)
2.5	2.5	1	0.15	0.1708	0.1203	1.0610	0.0260	0.0682	0.3283
				(0.0009)	(0.0007)	(0.0022)	(0.0002)	(0.0008)	(0.0514)
2.5	2.5	1	0.135	0.1691	0.1189	1.0567	0.0259	0.0663	0.2494
				(0.0004)	(0.0003)	(0.0009)	(0.0000)	(0.0003)	(0.0228)
2.5	2.5	1	0.12	0.1675	0.1176	1.0527	0.0262	0.0646	0.1661
				(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0008)

Table 7: Economic variables in the simulation with two agents for $\psi_i \approx 1$

The table gives the mean and the standard deviation (in brackets) of some central asset pricing moments and real values in the model with two investors. The parameters for the model are given in Table 2.
The *preference* column denotes whether the agents have logarithmic or recursive preferences. The *restr* column gives the value of $\bar{\sigma}_{FI}$ or denotes that there is no restriction in place (*unres*).

B Figures

B.1 Epstein-Zin utility with two investors

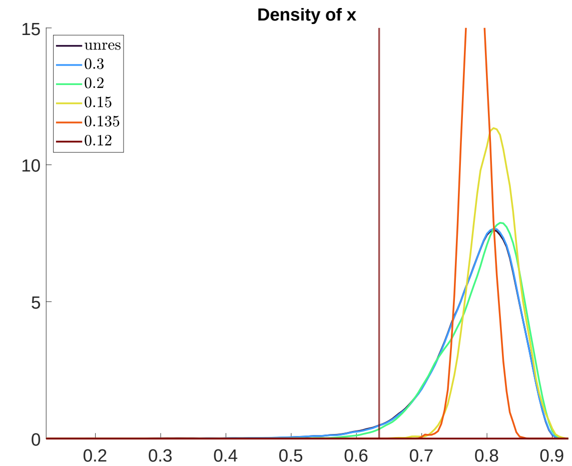


Figure 1: Density of x

The figure shows the density of x resulting from the Monte-Carlo Simulations of the model with Epstein-Zin preferences and two investors. The densities are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.

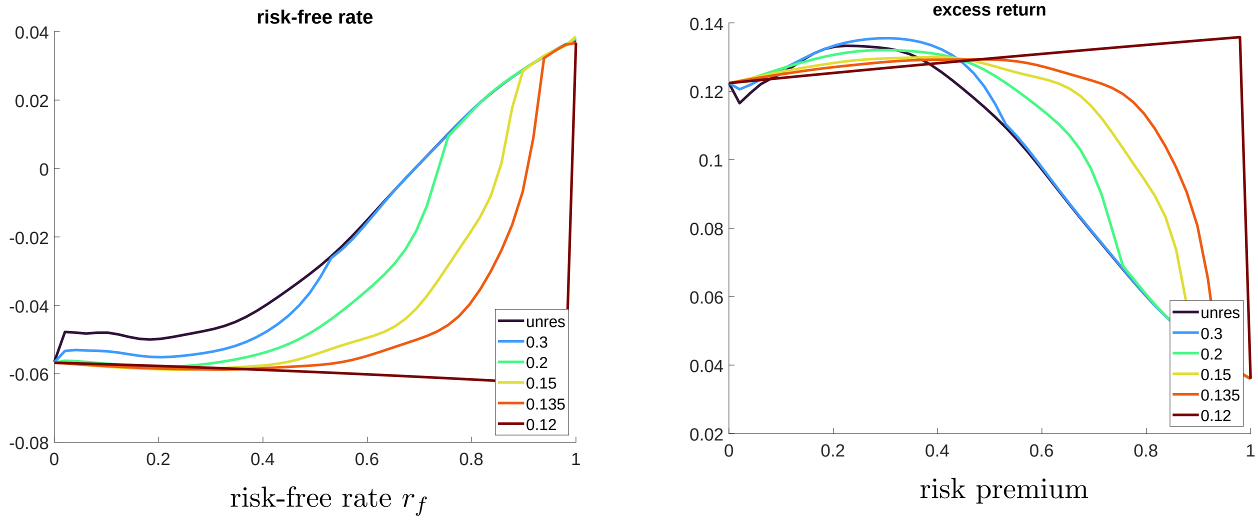


Figure 2: Risk-free rate and risk premium on consumption claim
The figure gives the risk-free rate (left graph) and the risk premium on the consumption claim (right graph) in the base case model with two investors with EZ preferences. The weights are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.

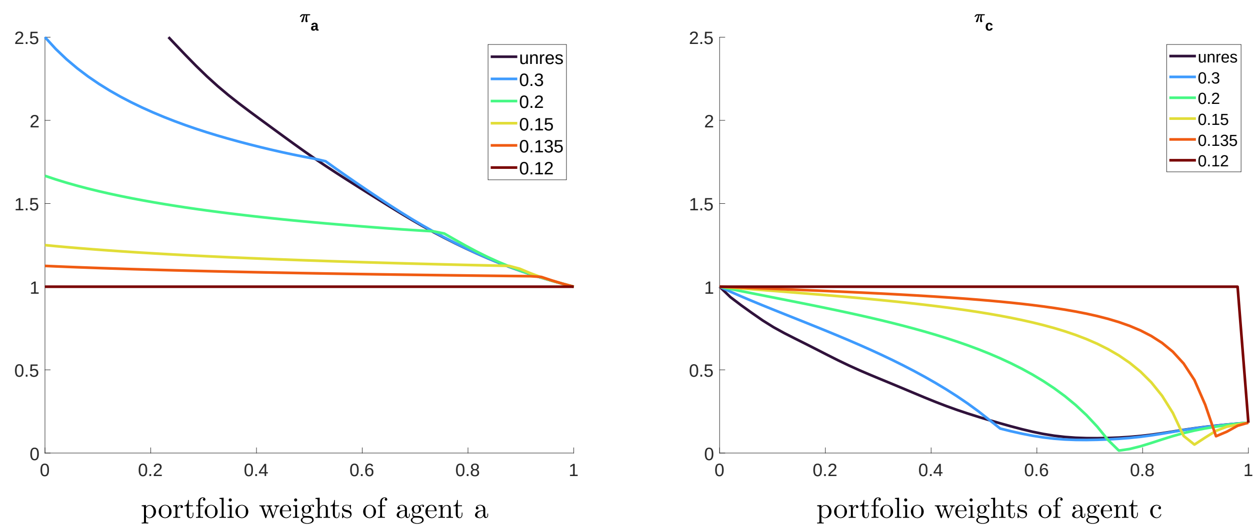
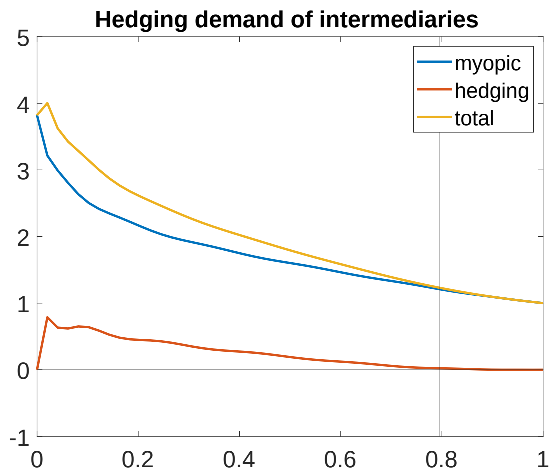
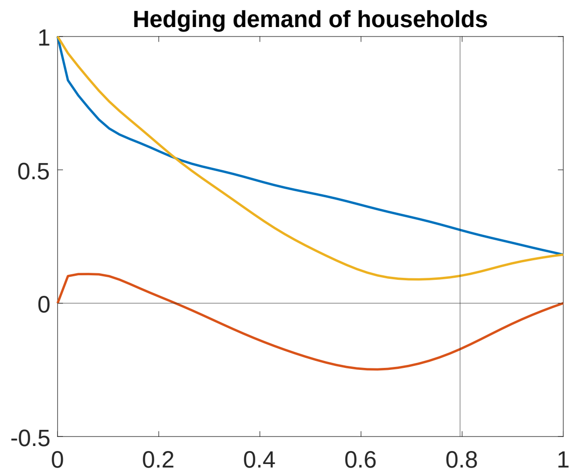


Figure 3: Portfolio weights
The figure gives the portfolio weights for the financial intermediary (left graph) and the household (right graph) in the base case model with two investors with EZ preferences. The weights are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.



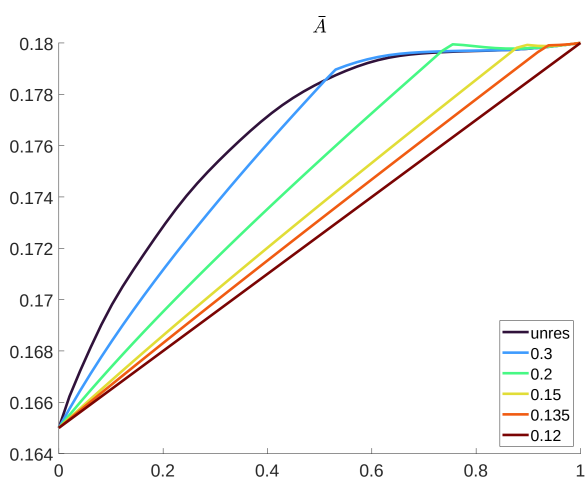
Demand decomposition of Intermediaries



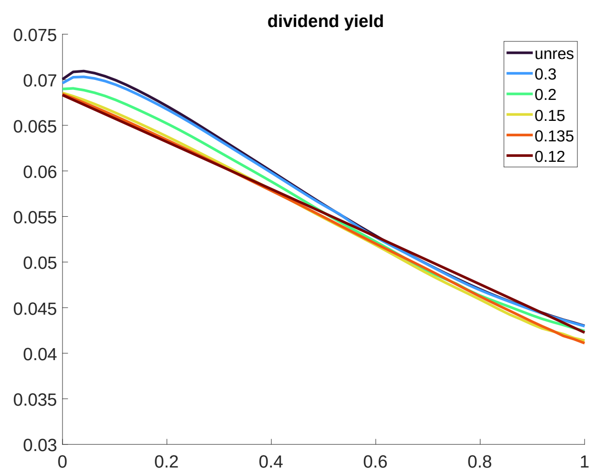
Demand decomposition of Households

Figure 4: Demand decomposition

The figure shows the demand composition for the financial intermediary (left graph) and the household (right graph) in the base case model with two investors with EZ preferences, without a restriction on the portfolio weights. The blue line describes the myopic demand, the red line the hedging demand and the yellow line the total demand. The parameters for the model are given in Table 2.



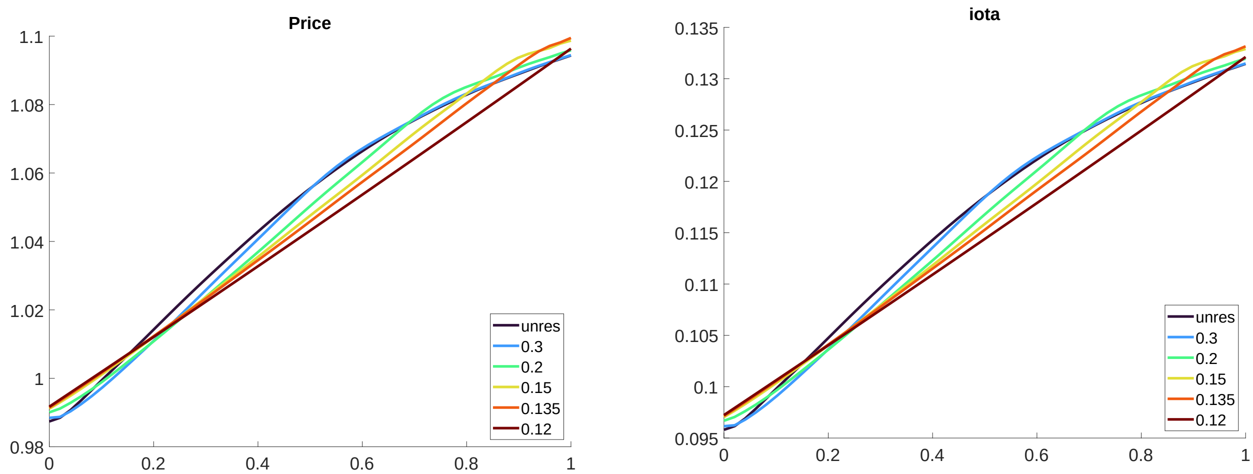
$\bar{A}_t$



Aggregate dividend yield $\frac{\bar{A}_t - \iota^* - \Phi(\iota^*)}{p_t}$

Figure 5: Average output and dividend yield

The figure gives the average output (left graph) and the dividend yield (right case) in the base case model with two investors with EZ preferences. The weights are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.



Price of capital

investment rate ι^*

Figure 6: Price of capital and investment rate

The figure gives the price of capital (left graph) and the investment rate (right graph) in the base case model with two investors with EZ preferences. The weights are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.

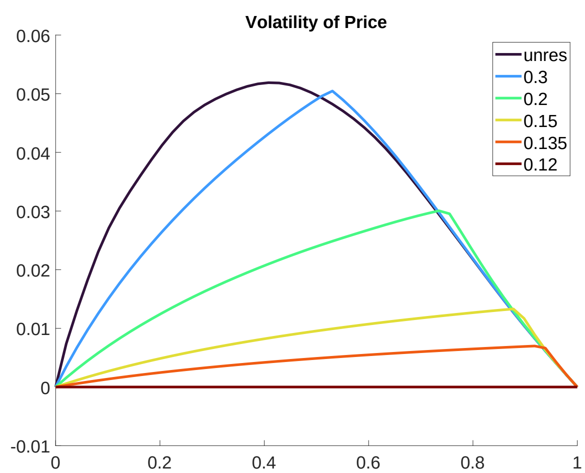


Figure 7: Volatility σ_p of price of capital

The figure gives the volatility of the price of capital in the base case model with two investors with EZ preferences. The weights are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.

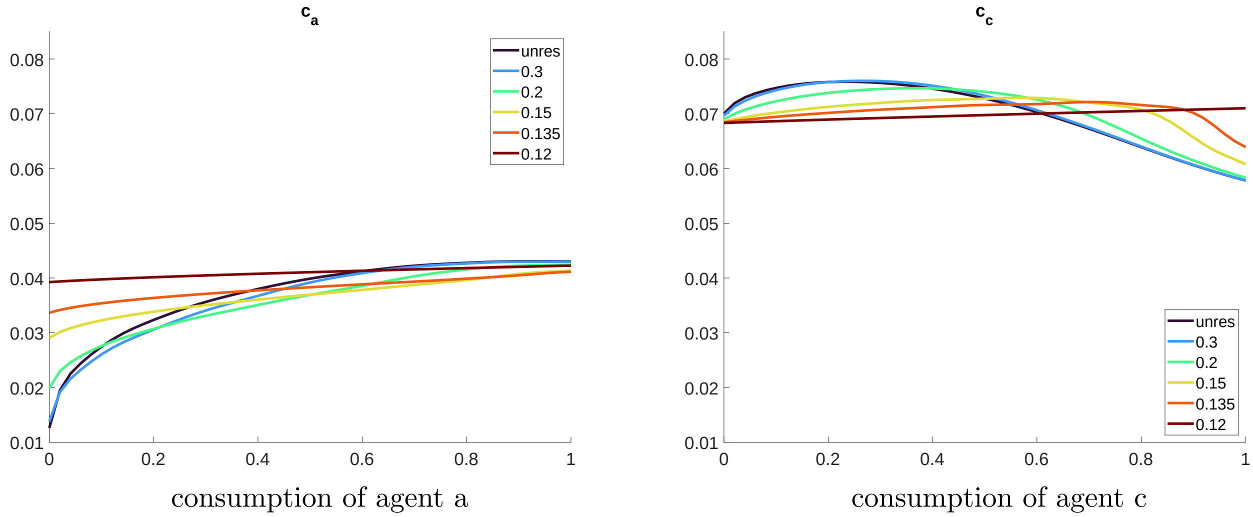


Figure 8: Consumption rate for the unrestricted and restricted cases
The figure gives the consumption rate for the financial intermediary (left graph) and the household (right graph) in the base case model with two investors with EZ preferences. The consumption rates are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.

B.2 Logarithmic utility with two investors

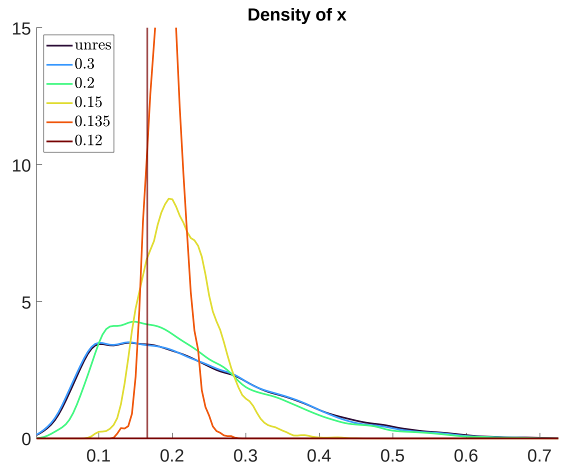
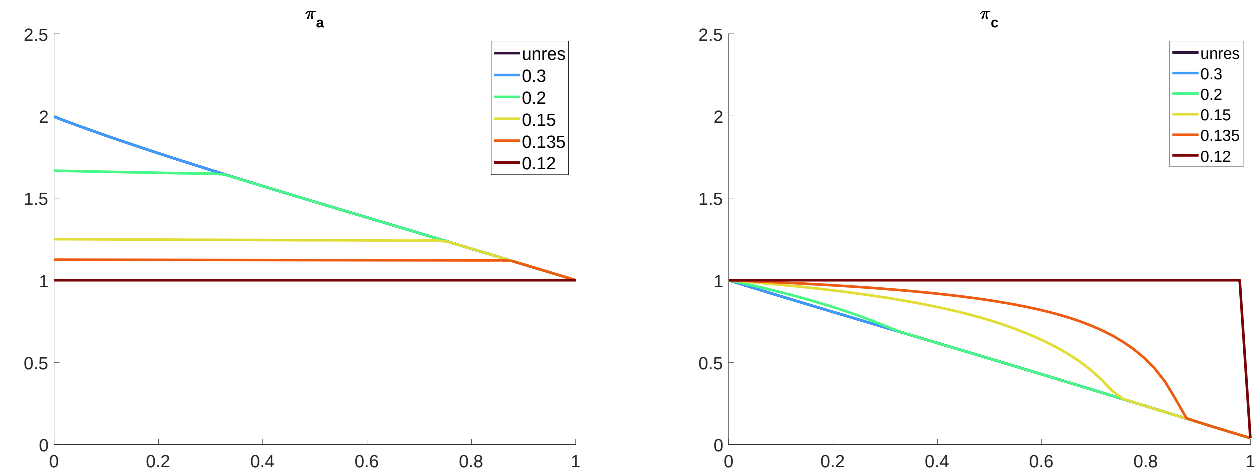


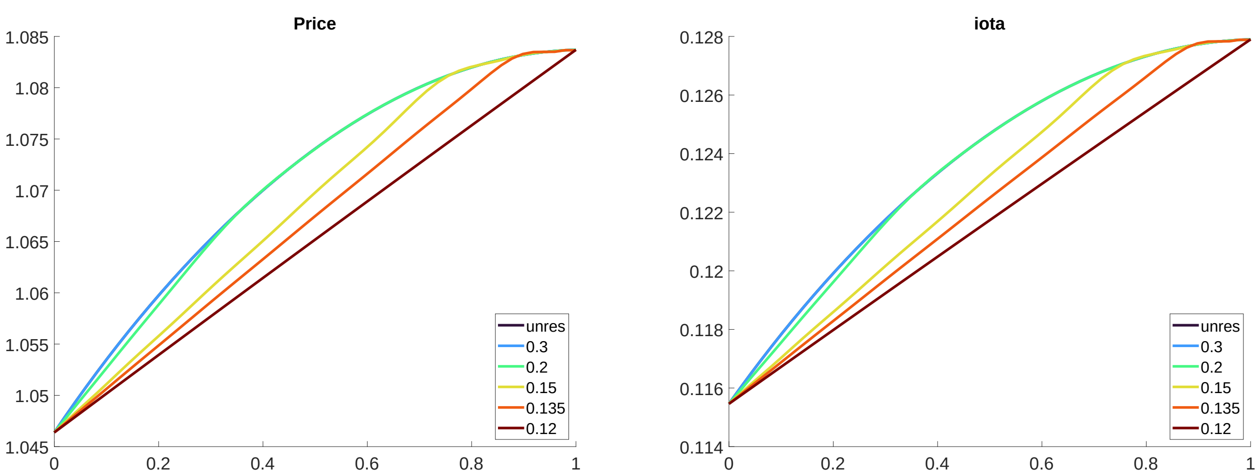
Figure 9: Density of x in the model with logarithmic preferences and two investors
The figure shows the density of x resulting from the Monte-Carlo Simulations of the model with logarithmic preferences and two investors. The densities are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.



portfolio weights of agent a

portfolio weights of agent b

Figure 10: Portfolio weights (log utility)
The figure gives the portfolio weights of the financial intermediary (left graph) and the household (right graph) in the model with two investors and log utility. The weights are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.



Price of capital

investment rate ι^*

Figure 11: Price of capital and investment rate ι (log utility)
The figure gives the price of capital (left graph) and the investment rate (right graph) in the model with two investors and log utility. The quantities are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.

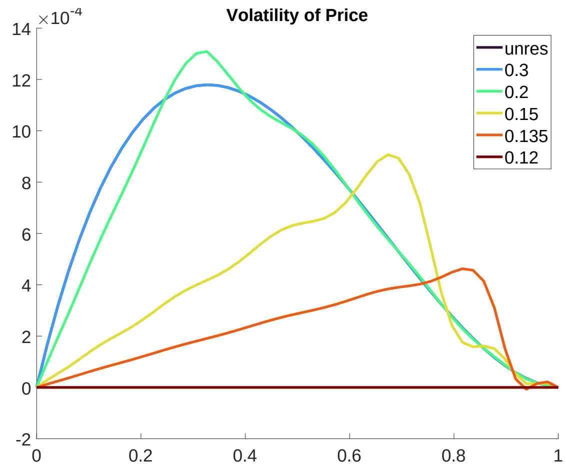
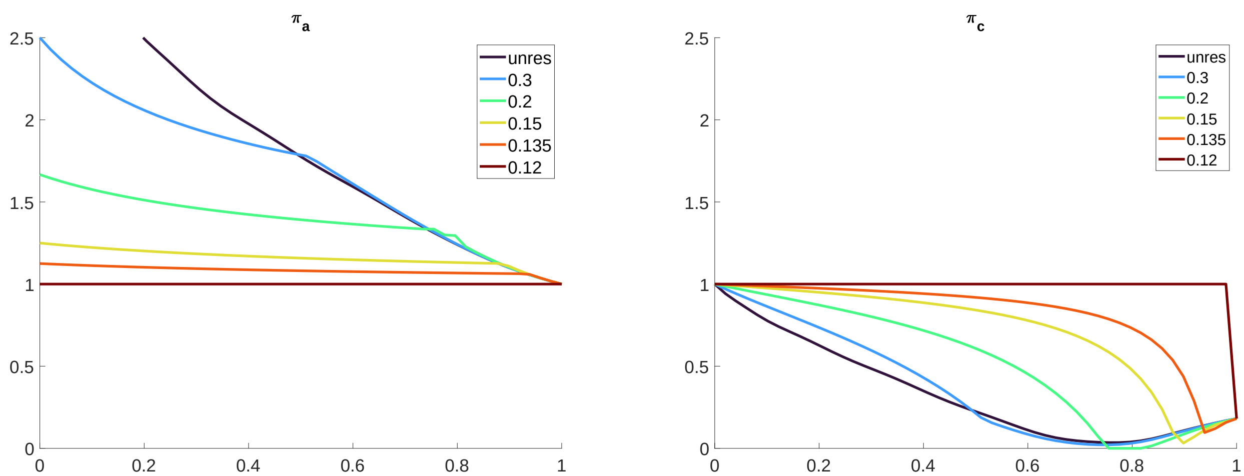


Figure 12: Volatility of price of capital σ_p (log preferences)
The figure gives the volatility of the price of capital in the model with two investors and log utility. The volatility is displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.

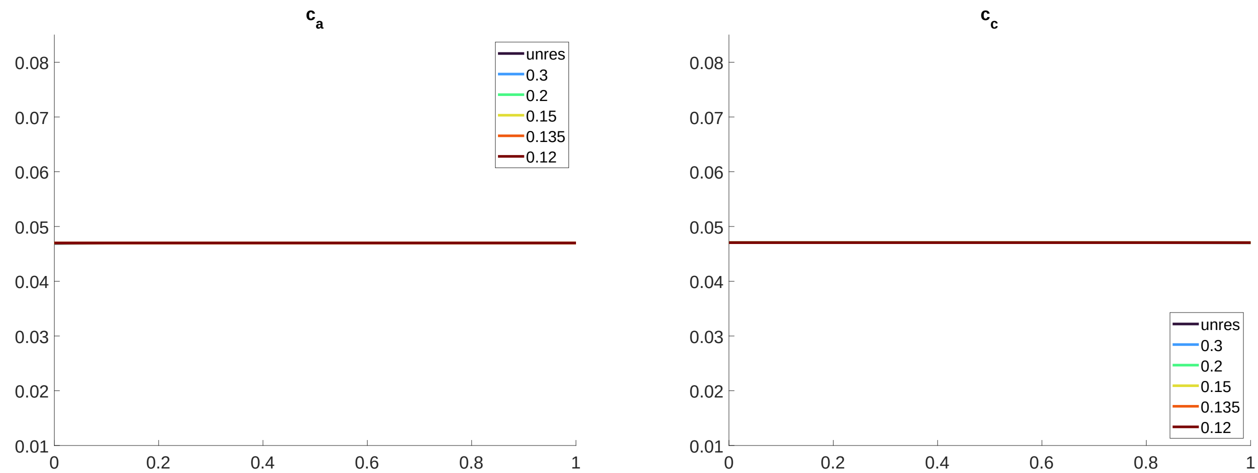
B.3 Epstein-Zin with two investors and $\psi_i \approx 1$



portfolio weights of agent a

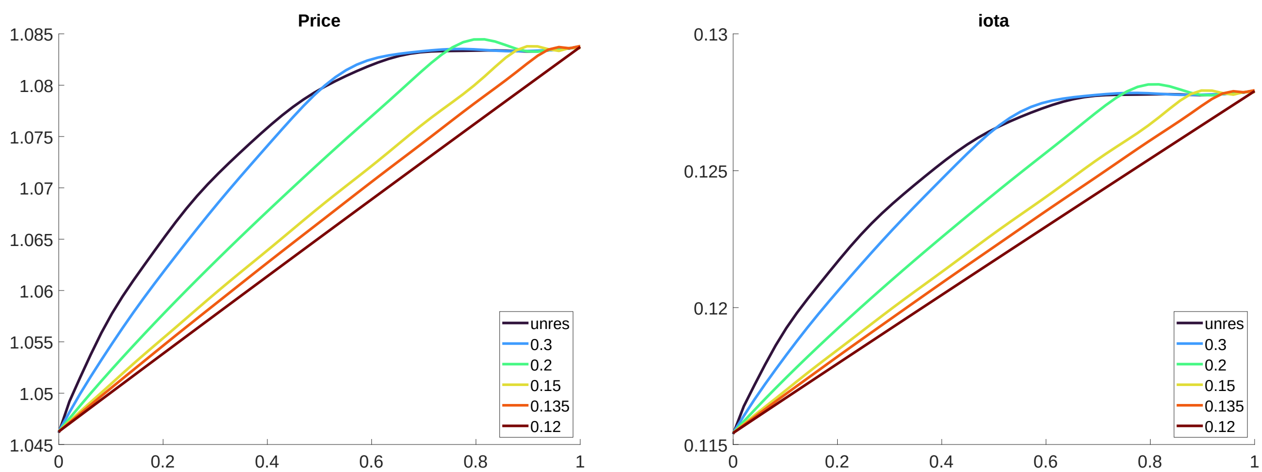
portfolio weights of agent c

Figure 13: Portfolio weights for EZ with $\psi \approx 1$
The figure gives the portfolio weights for the financial intermediary (left graph) and the household (right graph) in the model with two investors with EZ preferences where ψ is set to one. The weights are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.



consumption of agent a　　　consumption of agent c

Figure 14: Consumption rate for $\psi \approx 1$
The figure gives the consumption rates for the financial intermediary (left graph) and the household (right graph) in the model with two investors with EZ preferences where ψ is set to one. The rates are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.



Price of capital　　　investment rate ι^*

Figure 15: Price of capital and investment rate for $\psi \approx 1$
The figure gives the price of capital (left graph) and the investment rate (right graph) in the model with two investors with EZ preferences where ψ is set to one. The price and the investment rate are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.

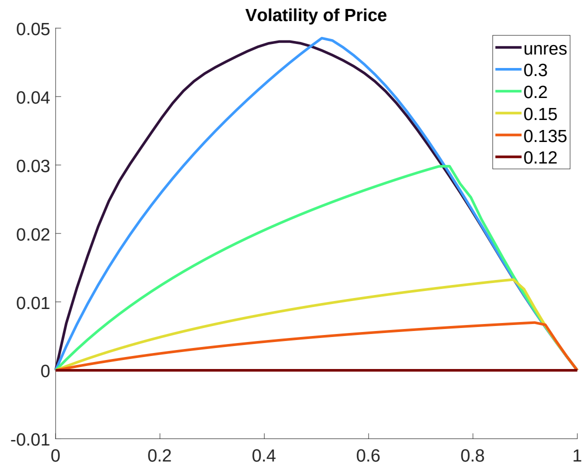


Figure 16: volatility of price σ_p for $\psi \approx 1$
The figure gives the volatility of the price of capital in the model with two investors with EZ preferences where ψ is set to one. The volatility is displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.

B.4 Epstein-Zin with two investors and $\gamma_a = \gamma_c = 2.5$

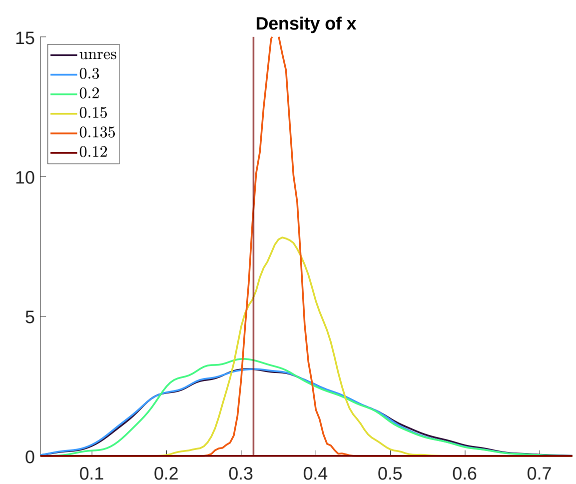
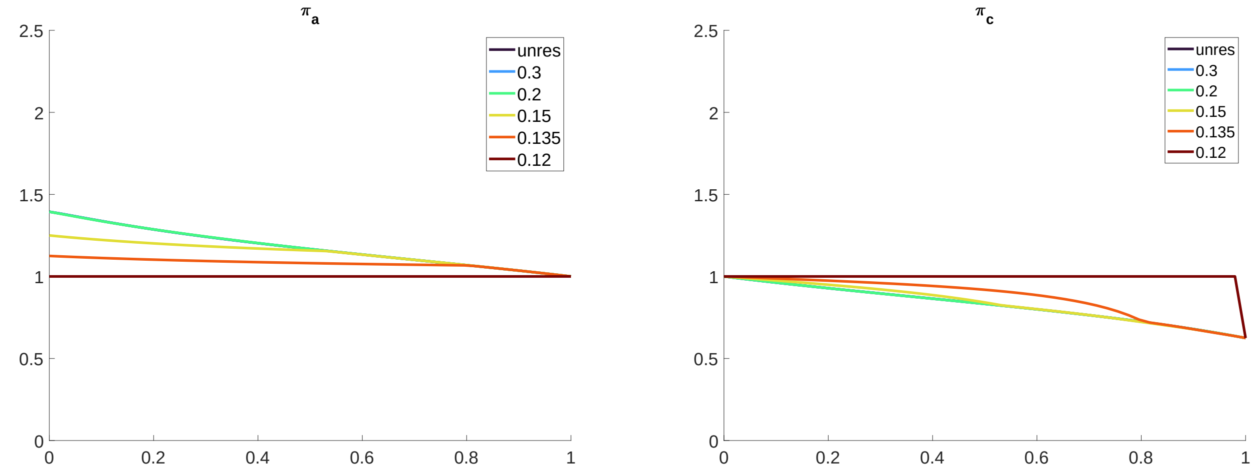


Figure 17: Density of x in the model with logarithmic preferences and two investors
The figure shows the density of x resulting from the Monte-Carlo Simulations of the model with EZ preferences where $\gamma_a = \gamma_c$ and two investors. The densities are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model (aside from the risk-aversions) are given in Table 2.

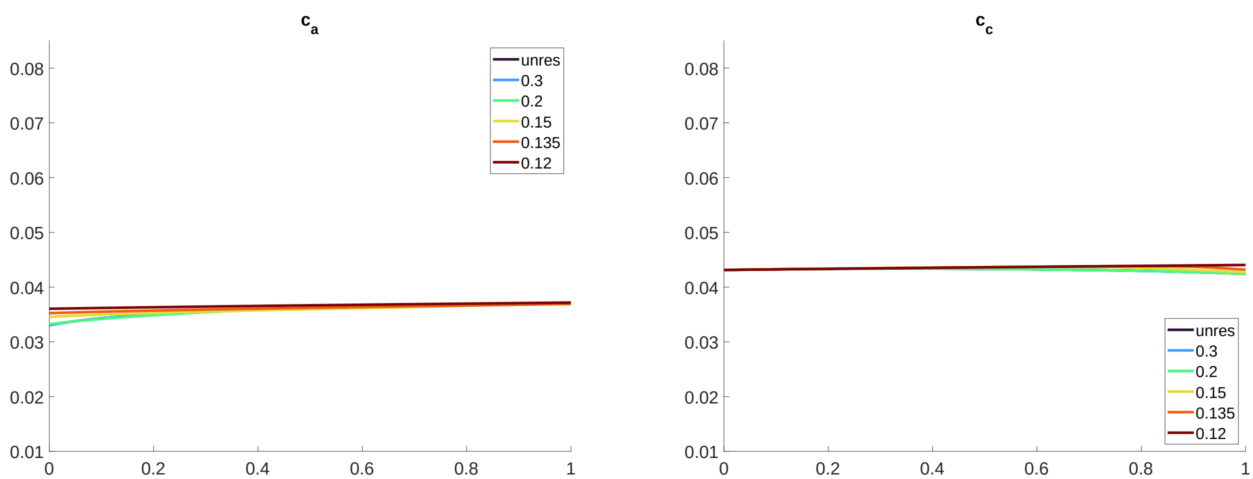


portfolio weights of agent a

portfolio weights of agent b

Figure 18: portfolio weights for EZ with $\gamma_a = \gamma_c = 2.5$

The figure gives the portfolio weights for the financial intermediary (left graph) and the household (right graph) in the model with two investors with EZ preferences where $\gamma_c = \gamma_a$. The weights are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.

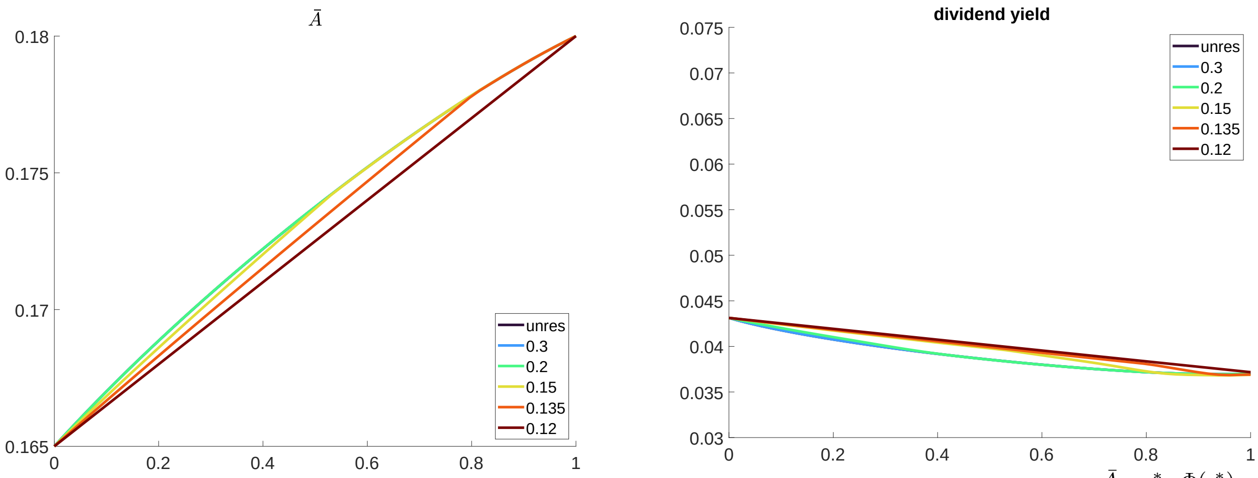


consumption of agent a

consumption of agent c

Figure 19: Consumption rate for EZ with $\gamma_a = \gamma_c = 2.5$

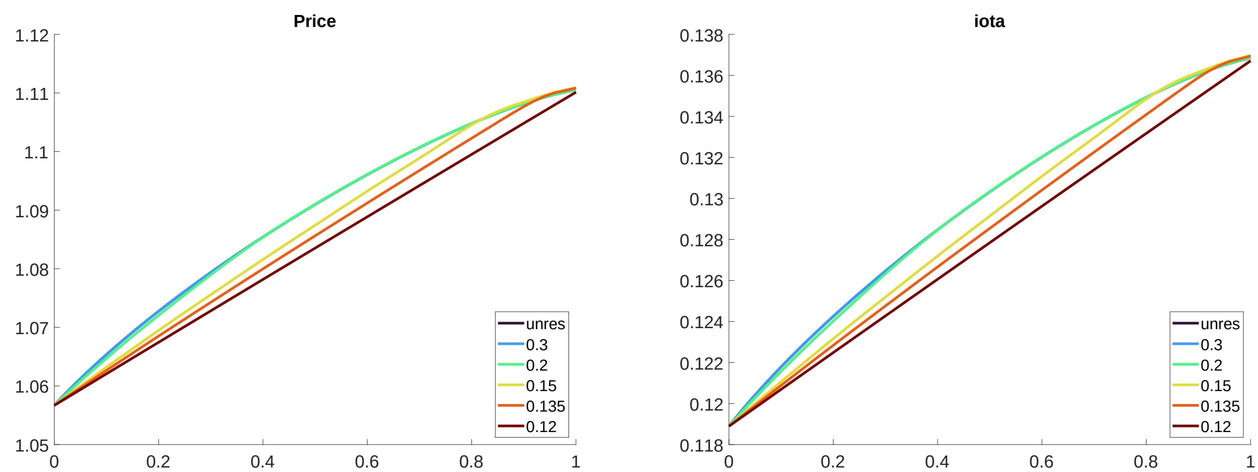
The figure gives the consumption rates for the financial intermediary (left graph) and the household (right graph) in the model with two investors with EZ preferences where $\gamma_c = \gamma_a$. The rates are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.



$\bar{A}_t$ Aggregate dividend yield $\frac{\bar{A}_t - \iota^* - \Phi(\iota^*)}{p_t}$

Figure 20: EZ with $\gamma_a = \gamma_c = 2.5$

The figure gives the average productivity (left graph) and the aggregate dividend yield (right graph) in the model with two investors with EZ preferences where $\gamma_c = \gamma_a$. The productivity and the dividend yield are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.



Price of capital investment rate ι^*

Figure 21: price of capital and investment rate for EZ with $\gamma_a = \gamma_c = 2.5$

The figure gives the price of capital (left graph) and the investment rate (right graph) in the model with two investors with EZ preferences where $\gamma_c = \gamma_a$. The price of capital and the investment rate are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.

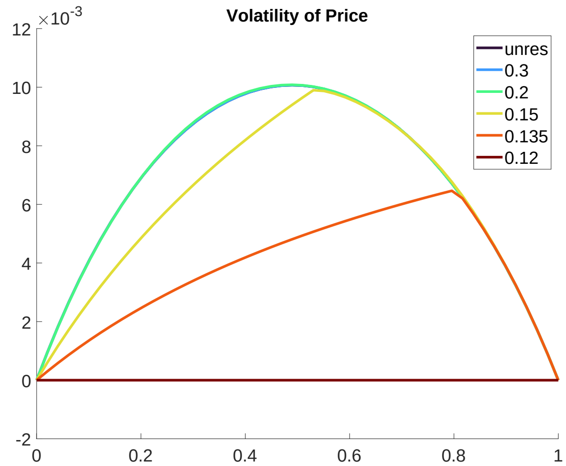
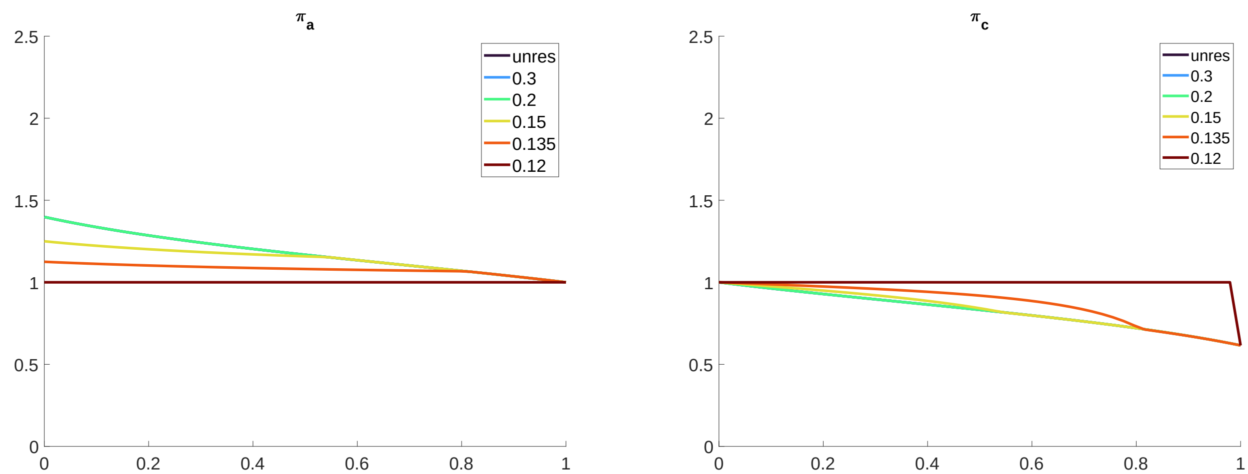


Figure 22: volatility of price σ_p for EZ with $\gamma_a = \gamma_c = 2.5$

The figure gives the volatility of the price of capital in the model with two investors with EZ preferences where $\gamma_c = \gamma_a$. The volatility is displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.

B.5 Epstein-Zin with two investors and $\psi_i \approx 1$ and $\gamma_a = \gamma_c = 2.5$

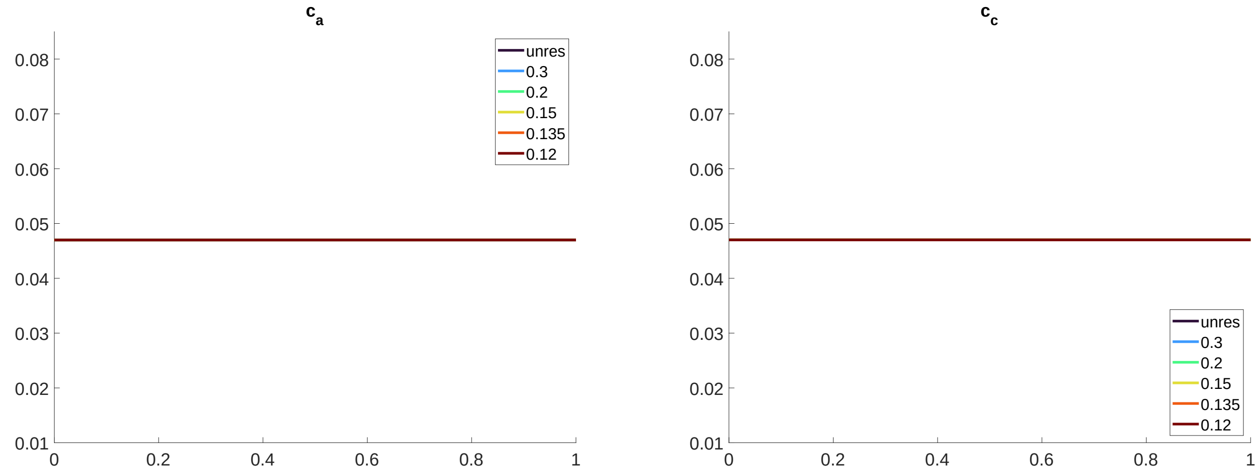


portfolio weights of agent a

portfolio weights of agent b

Figure 23: portfolio weights for EZ with $\psi_i \approx 1$ and $\gamma_a = \gamma_c = 2.5$

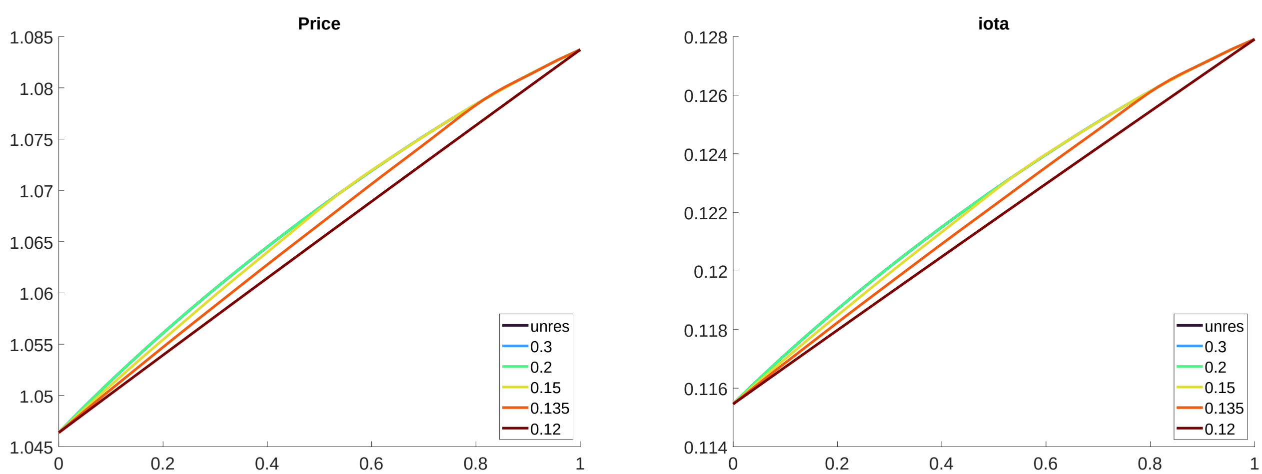
The figure gives the portfolio weights for the financial intermediary (left graph) and the household (right graph) in the model with two investors with EZ preferences where $\gamma_c = \gamma_a$ and $\psi = 1$. The weights are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.



consumption of agent a   consumption of agent c

Figure 24: Consumption rate for the unrestricted and restricted cases

The figure gives the consumption rates for the financial intermediary (left graph) and the household (right graph) in the model with two investors with EZ preferences where $\gamma_c = \gamma_a$ and $\psi = 1$. The consumption rates are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.

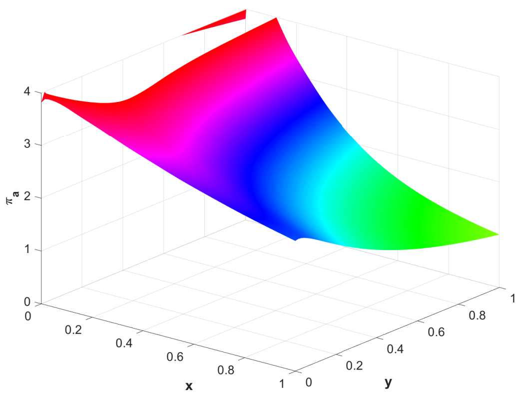


Price of capital   investment rate ι^*

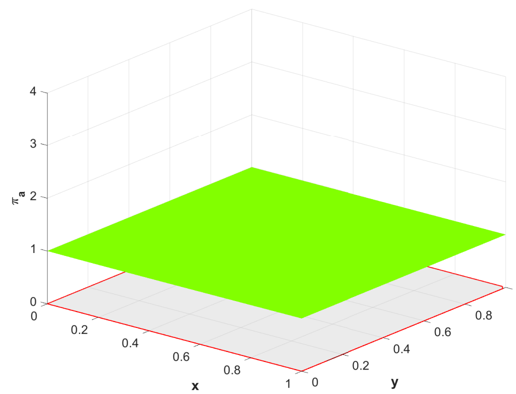
Figure 25: price of capital and investment rate for EZ with $\psi_i \approx 1$ and $\gamma_a = \gamma_c = 2.5$

The figure gives the price of capital (left graph) and the investment rate (right graph) in the model with two investors with EZ preferences where $\gamma_c = \gamma_a$ and $\psi = 1$. The rates are displayed for different values of the restriction $\bar{\sigma}_{FI}$, where the blue line refers to the unrestricted case and the brown line refers to the tightest restriction. The parameters for the model are given in Table 2.

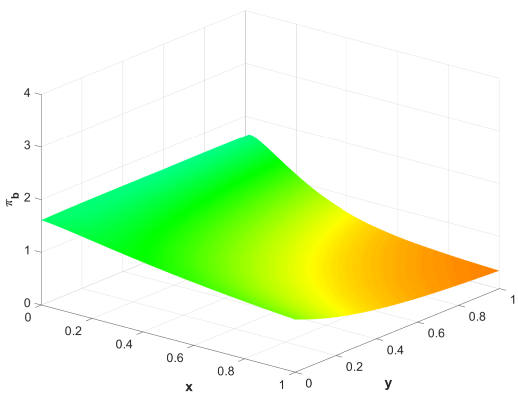
B.6 Epstein-Zin utility with three investors



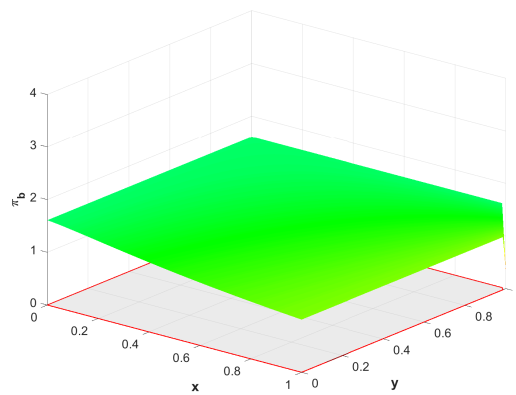
(a) Portfolio weights of type "a" without restriction



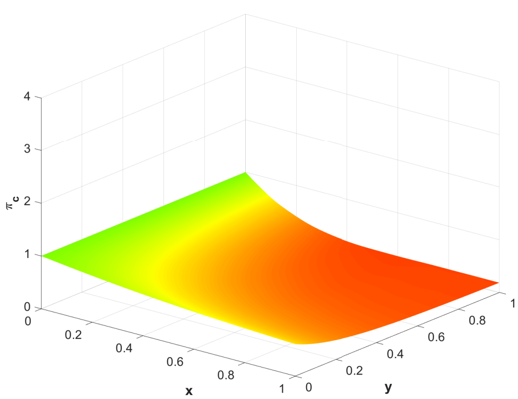
(b) Portfolio weights of type "a" with $\bar{\sigma}_{FI} = 0.12$



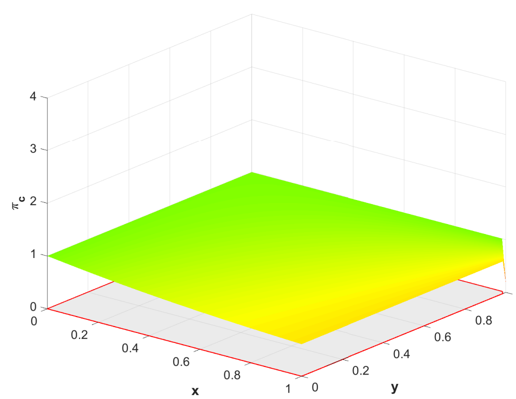
(c) Portfolio weights of type "b" without restriction



(d) Portfolio weights of type "b" with $\bar{\sigma}_{FI} = 0.12$



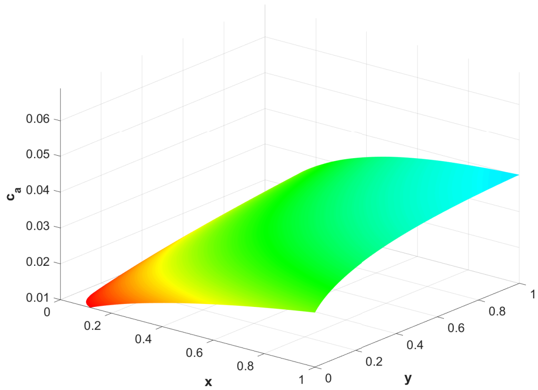
(e) Portfolio weights of type "c" without restriction



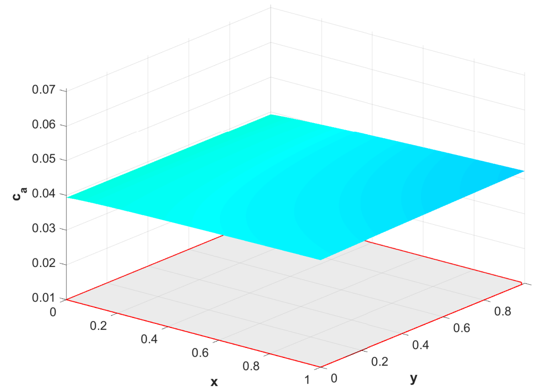
(f) Portfolio weights of type "c" with $\bar{\sigma}_{FI} = 0.12$

Figure 26: Portfolio weights in the EZ economy with three investors

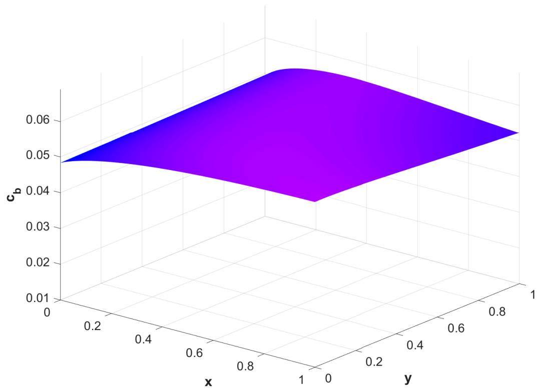
The figure gives the portfolio weights of agents a, agents b, and agents c in the unrestricted (left column) and for the restriction $\bar{\sigma}_{FI} = 0.12$ (right column). The shaded areas denote where the restriction is binding. The parameters for the model are given in Table 2; we assume Epstein-Zin preferences.



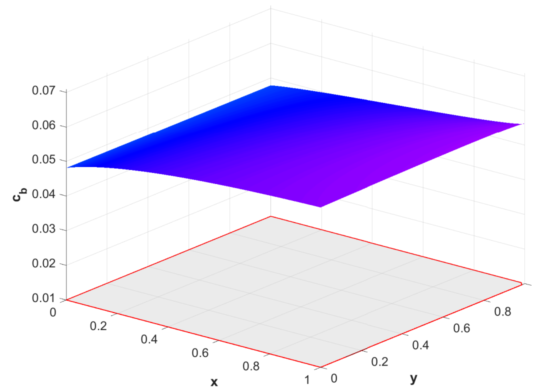
(a) Consumption of type "a" without restriction



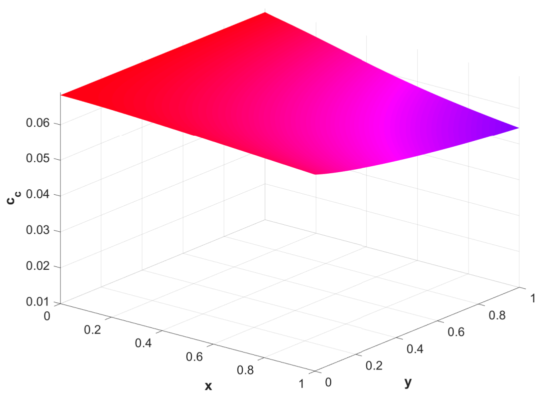
(b) Consumption of type "a" with $\bar{\sigma}_{FI} = 0.12$



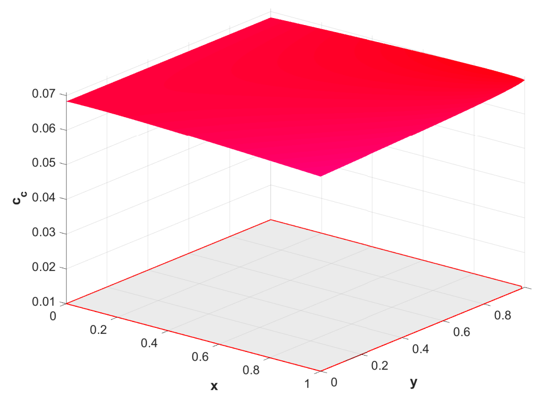
(c) Consumption of type "b" without restriction



(d) Consumption of type "b" with $\bar{\sigma}_{FI} = 0.12$



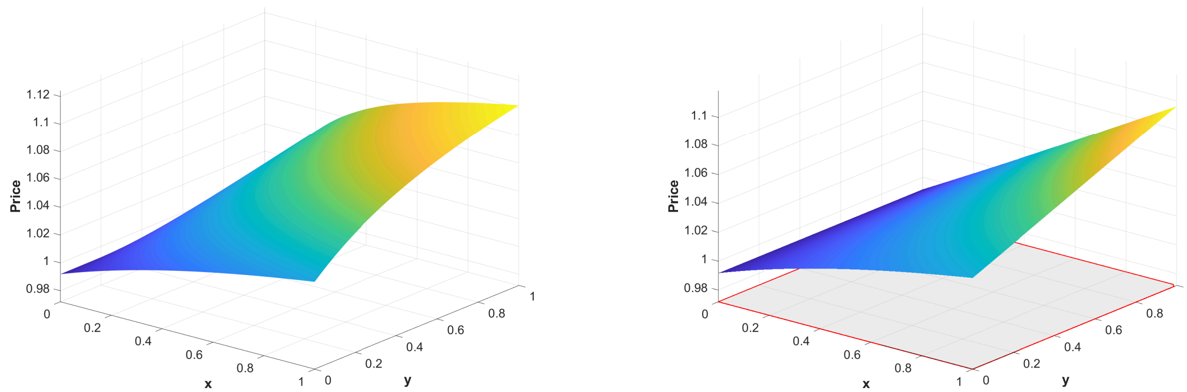
(e) Consumption of type "c" without restriction



(f) Consumption of type "c" with $\bar{\sigma}_{FI} = 0.12$

Figure 27: Consumption rates in the EZ economy with three investors

The figure gives the consumption rates of agents a, agents b, and agents c in the unrestricted (left column) and for the restriction $\bar{\sigma}_{FI} = 0.12$ (right column). The shaded areas denote where the restriction is binding. The parameters for the model are given in Table 2; we assume Epstein-Zin preferences.



(a) Price of capital without restriction

(b) Price of capital with $\bar{\sigma}_{FI} = 0.12$

Figure 28: Price of capital in the EZ Economy with three investors

The figure gives the price p_t of (installed) capital in the unrestricted (left column) and for the restriction $\bar{\sigma}_{FI} = 0.12$ (right column). The shaded areas denote where the restriction is binding. The parameters for the model are given in Table 2; we assume Epstein-Zin preferences.

C Calculations to solve the model with two investors

C.1 Deriving the law of motions of the state variables

We calculate the dynamics of the wealth share of the bank using Ito's Lemma:

$$
\begin{aligned}
dx_t =& d\frac{W_{a,t}}{W_{a,t}+W_{c,t}} \\
=& \frac{1}{W_{a,t}+W_{c,t}} dW_{a,t} - \frac{W_{a,t}}{(W_{a,t}+W_{c,t})^2} d(W_{a,t}+W_{c,t}) \\
& - \frac{1}{(W_{a,t}+W_{c,t})^2} dW_{a,t} d(W_{a,t}+W_{c,t}) + \tfrac{1}{2}\frac{2W_{a,t}}{(W_{a,t}+W_{c,t})^3}\left(d(W_{a,t}+W_{c,t})\right)^2
\end{aligned}
$$

We plug in the dynamics of the wealth (Equation (6)) and the market clearing condition for capital (Equation (17)). To account for the overlapping generations, we have to add the birth and death rates $\kappa(\bar{x}-x_t)$. Then, simplifying yields

$$
\begin{aligned}
dx_t =& \kappa(\bar{x}-x_t)dt \\
& + x_t(1-x_t)\Bigg[\big(\pi_{a,t}-\pi_{c,t}\big)\big(\iota_t^* - \delta + \mu_{p,t} + \sigma_K\sigma_{p,t} - r_t\big) \\
& \quad - \big(\pi_{a,t}-\pi_{c,t}\big)\big(x_t\pi_{a,t} + (1-x_t)\pi_{c,t}\big)\big(\sigma_K+\sigma_{p,t}\big)^2 \\
& \quad + \left(\pi_{a,t}\frac{A_a - \iota_t^* - \Phi(\iota_t^*)}{p_t} - \pi_{c,t}\frac{A_c - \iota_t^* - \Phi(\iota_t^*)}{p_t}\right) + c_{c,t} - c_{a,t}\Bigg]dt \\
& + \Big[x_t(1-x_t)(\pi_{a,t}-\pi_{c,t})(\sigma_K+\sigma_{p,t})\Big]dZ_t.
\end{aligned}
$$

C.2 Deriving the HJB

The HJB-approach implies to solve (6):

$$
\begin{aligned}
0 &= \max_{c_i,\pi_i} \left\{ f(C_i,V_i) + E(dV_i) \right\} \\
&= \max_{c_i,\pi_i} \Bigg\{ f(C_i,V_i) + \frac{\partial J}{\partial W} E\left[dW\right] + \frac{\partial V_i}{\partial x} E\left[dx\right] + \frac{1}{2}\left(\frac{\partial^2 V_i}{\partial W^2} E\left[(dW)^2\right] + \frac{\partial^2 V_i}{\partial x^2} E\left[(dx)^2\right] \right) \\
&\qquad + \frac{\partial^2 V_i}{\partial W \partial x} E\left[dW dx\right] \Bigg\} \\
&= \max_{c_i,\pi_i} \Bigg\{ f(C_i,V_i) + \frac{\partial V_i}{\partial W} W_{i,t} \left[r_t + \pi_{i,t} \left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota - \delta + \mu_p + \sigma_K \sigma_{p,t} - r_t \right) - \tfrac{C_{i,t}}{W_{i,t}} \right] \\
&\qquad + \frac{\partial V_i}{\partial x}\mu_x + \frac{1}{2}\left(\frac{\partial^2 V_i}{\partial W^2} W_{i,t}^2 \pi_{i,t}^2 (\sigma_K + \sigma_{p,t})^2 + \frac{\partial^2 V_i}{\partial x^2}\sigma_x^2 \right) + \frac{\partial^2 V_i}{\partial W \partial x} W_{i,t}\pi_{i,t}\sigma_x(\sigma_K + \sigma_{p,t}) \Bigg\}
\end{aligned}
$$

We use the usual guess for the indirect utility:

$$
V_i = \frac{W_i^{1-\gamma_i}}{1-\gamma_i} G_i(x)^{\frac{1-\gamma_i}{1-\psi_i}} \tag{30}
$$

where $G_i(x,K)$ describes the Value function of agent i. Plugging this in and simplifying gives

$$
\begin{aligned}
\rho_i + \kappa = \max_{\pi_{i,t},c_{i,t}} \Bigg\{ & G_{i,t} + (1 - \tfrac{1}{\psi_i}) \left(r_t + \pi_{i,t} \left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t} - r_t \right) - c_{i,t} \right) \\
& - \tfrac{1}{\psi_i} \frac{G_{i,t,x}}{G_{i,t}} \mu_{x,t} - \tfrac{1}{2}(1 - \tfrac{1}{\psi_i}) \gamma_i \pi_{i,t}^2 (\sigma_K + \sigma_{p,t})^2 \\
& - \tfrac{1}{2}\tfrac{1}{\psi_i} \left((\tfrac{1-\gamma_i}{1-\psi_i} - 1) \frac{(G_{i,t,x})^2}{G_{i,t}^2} + \frac{G_{i,t,xx}}{G_{i,t}} \right) \sigma_{x,t}^2 - \frac{1-\gamma_i}{\psi_i} \frac{G_{i,t,x}}{G_{i,t}} \pi_{i,t} \sigma_{x,t} (\sigma_K + \sigma_{p,t}) \Bigg\}.
\end{aligned}
$$

The first-order condition gives the optimality of the consumption:

$$
\frac{\partial f}{\partial C} = \frac{\partial V_i}{\partial W} \tag{31}
$$

Deriving the Epstein-Zinn aggregator

$$f(C_i,V_i) = \frac{1-\gamma_i}{1-\frac{1}{\psi_i}} V_i \left[\left(\frac{C_i}{[(1-\gamma_i)V_i]^{\frac{1}{1-\gamma_i}}} \right)^{1-\frac{1}{\psi_i}} - \rho_i \right] \tag{32}$$

$$= \frac{C_i^{1-\frac{1}{\psi_i}}}{1-\frac{1}{\psi_i}} [(1-\gamma_i)V_i]^{1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}} - \frac{1-\gamma_i}{1-\frac{1}{\psi_i}} V_i \rho_i \tag{33}$$

with respect to the consumption yields

$$f_C(C_i,V_i) = [(1-\gamma_i)V_i]^{1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}} C_i^{-\frac{1}{\psi_i}}$$

and the derivative with respect to W_i of our guess in Equation (56) for the indirect utility is

$$\frac{\partial V_i}{\partial W} = W_i^{-\gamma_i} G_i(x)^{\frac{1-\gamma_i}{1-\psi_i}}. \tag{34}$$

Hence we can plug in everything in (31) and simplify to get the optimal consumption:

$$\left[(1-\gamma_i) \frac{W_i^{1-\gamma_i}}{1-\gamma_i} G_i(x)^{\frac{1-\gamma_i}{1-\psi_i}} \right]^{1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}} C_i^{-\frac{1}{\psi_i}} = W^{-\gamma_i} G_i(x)^{\frac{1-\gamma_i}{1-\psi_i}} \tag{35}$$

$$C_i = W_i G_i(x) \tag{36}$$

Hence, the value function is also the consumption wealth ratio. The aggregator becomes

$$\begin{aligned} f(C_i,V_i) =& \frac{1}{1-\frac{1}{\psi_i}} (W_i G_i(x))^{1-\frac{1}{\psi_i}} W_i^{(1-\gamma_i)\left(1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}\right)} G_i(x)^{\frac{1-\gamma_i}{1-\psi_i}\left(1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}\right)} \\ &- \frac{\rho_i}{1-\frac{1}{\psi_i}} W_i^{1-\gamma_i} G_i(x)^{\frac{1-\gamma_i}{1-\psi_i}} \\ =& \frac{1}{1-\frac{1}{\psi_i}} W_i^{1-\gamma_i} G_i(x)^{\frac{1-\gamma_i}{1-\psi_i}} (G_i(x) - \rho_i) \end{aligned}$$

Plugging this in and simplifying, we get

$$\rho_i + \kappa = \max_{\pi_{i,t}} \Bigg\{ G_i + (1 - \tfrac{1}{\psi_i}) \left(r_t + \pi_i \left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota - \delta + \mu_p + \sigma_K \sigma_{p,t} - r_t \right) - \tfrac{C_i}{W_i} \right)$$
$$- \tfrac{1}{\psi_i} \frac{G_{i,x}}{G_i} \mu_x - \tfrac{1}{2}(1 - \tfrac{1}{\psi_i}) \gamma_i \pi_i^2 (\sigma_K + \sigma_{p,t})^2 - \tfrac{1}{2} \tfrac{1}{\psi_i} \left((\tfrac{1-\gamma_i}{1-\psi_i} - 1) \frac{(G_{i,x})^2}{G_i^2} + \frac{G_{i,xx}}{G_i} \right) \sigma_x^2$$
$$- \frac{1 - \gamma_i}{\psi_i} \frac{G_{i,x}}{G_i} \pi_i \sigma_x (\sigma_K + \sigma_{p,t}) \Bigg\} \tag{37}$$

Deriving this equation with respect to $\pi_{i,t}$ and solving for that unknown gives the first-order condition for the portfolio weights:

$$\pi_{i,t} = \frac{1}{\gamma_i} \frac{\left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota - \delta + \mu_p + \sigma_K \sigma_t^p - r_t \right)}{(\sigma_K + \sigma_t^p)^2} + \frac{1}{\gamma_i} \frac{1 - \gamma_i}{1 - \psi_i} \left(\frac{G_{i,x}}{G_i} \frac{(\sigma_K + \sigma_t^p) \sigma_x}{(\sigma_K + \sigma_t^p)^2} \right) \tag{38}$$

C.3 Certainty-equivalent consumption

In order to compare the well-fare of the agents in the different model specifications, we calculate the certainty-equivalent consumption.

The certainty-equivalent (CE_i) consumption is defined as the constant consumption level that gives the same lifetime utility as the actual consumption stream[17]. Therefore, we need to solve

$$V_i(W_i, x) = \frac{CE_i^{1-\gamma_i}}{1 - \gamma_i} \tag{39}$$

Plugging in the guess for the indirect utility (Equation (30)) and simplifying gives

$$CE_i = W_i G_i(x)^{\frac{1}{1-\psi_i}}. \tag{40}$$

[17] We can *ignore* the continuation value and the impact of the IES in this case because there are no consumption surprises and no intertemporal substitution of the consumption is constant.

D Calculations to solve the model with three investors

D.1 Description of the three-agent model

Agents. The economy is populated by three agents indexed by

$$i \in \{a, b, c\}, \tag{41}$$

where type a agents correspond to *Funds*, type b agents to *Bank Holding Companies* and type c agents to *Households*. So the type a and type b agents make up the *bank* (type a) in the two-agent model. Again, agents exhibit the following ordering for the *risk aversion* parameter γ_i such that

$$\gamma_a < \gamma_b < \gamma_c. \tag{42}$$

Again, all agents exhibit the same *intertemporal elasticity of substitution*, i.e. $\psi_a = \psi_b = \psi_c = \psi$.

As before, each agent has a different skill level, such that agent a has the highest skill, agent b is similarly or less skilled than agent a and agent c has the lowest knowledge. This is again characterized by the constants

$$A_a \geq A_b > A_c. \tag{43}$$

The policy functions and optimization problem of each individual agent are similar to before (see (6)).

Production Sector. The production sector (described in section 2.2) remains the same, with the only exception being equation (11), which becomes

$$W_{a,t} + W_{b,t} + W_{c,t} = p_t K_t, \tag{44}$$

and equation (15), which becomes

$$\overline{A}_t = \pi_{a,t} x_t y_t A_a + \pi_{b,t} x_t (1 - y_t) A_b + \pi_{c,t} (1 - x_t) A_c. \tag{45}$$

State Variables. Analogously to above, each agent is fully characterized by the joint *wealth* levels $W_{a,t}, W_{b,t}$ and $W_{c,t}$. Agents jointly interact through market clearing so that they need to know the decisions of all other agents as via market clearing this will affect their own decisions.

The state variables are reformulated to the following wealth shares

$$x_t \equiv \frac{W_{a,t} + W_{b,t}}{W_{a,t} + W_{b,t} + W_{c,t}} \qquad\qquad y_t \equiv \frac{W_{a,t}}{W_{a,t} + W_{b,t}} \tag{46}$$

such that x_t is the overall wealth share of the a and b agents, which we can think of as the share of the *financial sector* of the overall economy, and y_t is the share of the type a agents within the financial sector. Thus, the wealth share of type a agents is given by $x_t y_t$, the wealth share of type b agents by $x_t(1 - y_t)$ and that of type c agents by $(1 - x_t)$. The Laws of motion of the state variables will be shown below.

Applying Ito's Lemma on (46) and using (6) and (16) gives that

$$\begin{aligned} dx_t =& \kappa(\bar{x} - x_t)dt \qquad\qquad (47) \\ &+ x_t(1 - x_t)\Bigg[\left(\iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t} - r_t - (\sigma_K + \sigma_{p,t})^2\right) \left(y_t \pi_{a,t} + (1 - y_t)\pi_{b,t} - \pi_{c,t}\right) \\ &\quad - \left(y_t c_{a,t} + (1 - y_t) c_{b,t} - c_{c,t}\right) \\ &\quad + \left(y_t \pi_{a,t} \frac{A_a - \iota_t^* - \Phi(\iota_t^*)}{p_t} + (1 - y_t)\pi_{b,t} \frac{A_b - \iota_t^* - \Phi(\iota_t^*)}{p_t} - \pi_{c,t} \frac{A_c - \iota_t^* - \Phi(\iota_t^*)}{p_t} \right) \Bigg] dt \\ &+ x_t(1 - x_t)\left[y_t \pi_{a,t} + (1 - y_t)\pi_{b,t} - \pi_{c,t}\right](\sigma_K + \sigma_{p,t}) dZ_t, \end{aligned}$$

where the drift coefficient is denoted by $\mu_{x,t}$ and the diffusion coefficient by $\sigma_{x,t}$.

Analogously, for y_t it follows that

$$
\begin{aligned}
dy_t =& \kappa(\bar{y} - y_t)dt \\
&+ y_t \Bigg[(1-y_t) \Big[\big(\pi_{a,t} - \pi_{b,t}\big)\big(\iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t} - r_t\big) \\
&\quad - \big(\pi_{a,t} - \pi_{b,t}\big)\big(y_t \pi_{a,t} + (1-y_t)\pi_{b,t}\big)\big(\sigma_K + \sigma_{p,t}\big)^2 \Big] \\
&\quad + (1-y_t) \left(\pi_{a,t} \frac{A_a - \iota_t^* - \Phi(\iota_t^*)}{p_t} - \pi_{b,t} \frac{A_b - \iota_t^* - \Phi(\iota_t^*)}{p_t} \right) + c_{b,t} - c_{a,t} \Bigg] dt \\
&+ \Big[y_t(1-y_t)(\pi_{a,t} - \pi_{b,t})(\sigma_K + \sigma_{p,t}) \Big] dZ_t.
\end{aligned} \tag{48}
$$

Likewise, the drift coefficient is denoted by $\mu_{y,t}$ and the diffusion coefficient by $\sigma_{y,t}$. The derivations of dx_t and dy_t are in the appendix in section D.2.

However, in x_t and y_t the drift ($\mu_{p,t}$) and diffusion ($\sigma_{p,t}$) coefficient of the price p_t is still unknown. As everything, apart from the constants, is a function of the state variables x_t and y_t, we use this fact to obtain the following for $\mu_{p,t}$ and $\sigma_{p,t}$

$$
\begin{aligned}
\frac{dp_t(x,y)}{p_t(x,y)} =& \underbrace{\left(\frac{p_{t,x}}{p}\mu_{x,t} + \frac{p_{t,y}}{p_t}\mu_{y,t} + \frac{p_{t,xy}}{p_t}\sigma_{x,t}\sigma_{y,t} + 0.5\frac{p_{t,xx}}{p_t}\sigma_{x,t}^2 + 0.5\frac{p_{t,yy}}{p_t}\sigma_{y,t}^2 \right)}_{=\mu_{p,t}} dt \\
&+ \underbrace{\left(\frac{p_{t,x}}{p_t}\sigma_{x,t} + \frac{p_{t,y}}{p_t}\sigma_{y,t} \right)}_{=\sigma_{p,t}} dZ_t.
\end{aligned} \tag{49}
$$

Market Clearing Analogously to the two-agent model, we can write the market clearing conditions in terms of the state variables x_t and y_t. For the capital market, the condition becomes (compare with Equation (17))

$$
\pi_{a,t}x_t y_t + \pi_{b,t}x_t(1-y_t) + \pi_{c,t}(1-x_t) = 1. \tag{50}
$$

The bond market clears if the following holds (compare with Equation (18))

$$(1-\pi_{a,t})x_t y_t + (1-\pi_{b,t})x_t(1-y_t) + (1-\pi_{c,t})(1-x_t) = 0. \tag{51}$$

Finally, the Goods market clears for the following condition

$$\overline{A}_t = p_t\big[c_{a,t}x_t y_t + c_{b,t}x_t(1-y_t) + c_{c,t}(1-x_t)\big] + \iota^* + \Phi(\iota^*). \tag{52}$$

Hamilton Jacob Bellman equations. Using (47) and (48) on (6), the HJB for each agent is given by the following where $G_{i,t} \equiv G_i(t, x_t, y_t)$ is the Value Function of agent i (The exact derivation is in the appendix in section D.3)

$$\begin{aligned}
\rho_i + \kappa = \max_{\pi_{i,t}, c_{i,t}} \Bigg\{ & G_{i,t} + (1-\tfrac{1}{\psi_i})\left(r_t + \pi_{i,t}\left(\frac{A_i - \iota_t^* - \Phi(\iota_t^*)}{p_t} + \iota_t^* - \delta + \mu_{p,t} + \sigma_K\sigma_{p,t} - r_t\right) - c_{i,t}\right) \\
& - \tfrac{1}{\psi_i}\left(\frac{G_{i,t,x}}{G_{i,t}}\mu_{x,t} + \frac{G_{i,t,y}}{G_{i,t}}\mu_{y,t}\right) - \tfrac{1}{2}(1-\tfrac{1}{\psi_i})\gamma_i\pi_{i,t}^2(\sigma_K + \sigma_{p,t})^2 \\
& - \tfrac{1}{2}\tfrac{1}{\psi_i}\left((\tfrac{1-\gamma_i}{1-\psi_i} - 1)\frac{(G_{i,t,x})^2}{G_{i,t}^2} + \frac{G_{i,t,xx}}{G_{i,t}}\right)\sigma_{x,t}^2 \\
& - \tfrac{1}{2}\tfrac{1}{\psi_i}\left((\tfrac{1-\gamma_i}{1-\psi_i} - 1)\frac{(G_{i,t,y})^2}{G_{i,t}^2} + \frac{G_{i,t,yy}}{G_{i,t}}\right)\sigma_{y,t}^2 \\
& - \tfrac{1}{\psi_i}\left((\tfrac{1-\gamma_i}{1-\psi_i} - 1)\frac{G_{i,t,x}G_{i,t,y}}{G_{i,t}^2} + \frac{G_{i,t,xy}}{G_{i,t}}\right)\sigma_{x,t}\sigma_{y,t} \\
& - \frac{1-\gamma_i}{\psi_i}\frac{G_{i,t,x}}{G_{i,t}}\pi_{i,t}\sigma_{x,t}(\sigma_K + \sigma_{p,t}) - \frac{1-\gamma_i}{\psi_i}\frac{G_{i,t,y}}{G_{i,t}}\pi_{i,t}\sigma_{y,t}(\sigma_K + \sigma_{p,t}) \Bigg\},
\end{aligned} \tag{53}$$

and optimal investment is substituted from (13).

The first-order condition for the portfolio weights gives the following form:

$$\pi_{i,t} = \frac{1}{\gamma_i}\frac{\left(\frac{A_i - \Phi(\iota)}{p_t} + \iota - \delta + \mu_p + \sigma_K\sigma_t^p - r_t\right)}{(\sigma_K + \sigma_t^p)^2} + \frac{1}{\gamma_i}\frac{1-\gamma_i}{1-\psi_i}\left(\frac{G_{i,x}}{G_i}\frac{(\sigma_K + \sigma_t^p)\sigma_x}{(\sigma_K + \sigma_t^p)^2} + \frac{G_{i,y}}{G_i}\frac{(\sigma_K + \sigma_t^p)\sigma_y}{(\sigma_K + \sigma_t^p)^2}\right) \tag{54}$$

The first term is the myopic demand for the capital which is the same for investors with logarithmic preferences. The second term is the hedging demand for changes in the state variables.

D.2 Deriving the law of motions of the state variables

We calculate the dynamics of the wealth share of intermediaries using Ito's Lemma:

$$\begin{aligned}
dx_t =& d\,\frac{W_{a,t}+W_{b,t}}{W_{a,t}+W_{b,t}+W_{c,t}} \\
=& \frac{1}{W_{a,t}+W_{b,t}+W_{c,t}} d(W_{a,t}+W_{b,t}) - \frac{W_{a,t}+W_{b,t}}{(W_{a,t}+W_{b,t}+W_{c,t})^2} d(W_{a,t}+W_{b,t}+W_{c,t}) \\
&- \frac{1}{(W_{a,t}+W_{b,t}+W_{c,t})^2} d(W_{a,t}+W_{b,t}+W_{c,t})\, d(W_{a,t}+W_{b,t}) \\
&+ \tfrac{1}{2} \frac{2(W_{a,t}+W_{b,t})}{(W_{a,t}+W_{b,t}+W_{c,t})^3} \left((W_{a,t}+W_{b,t}+W_{c,t})\right)^2
\end{aligned}$$

The next step is to plugg in the wealth dynamics (i.e., the budget constraint in Equation (6)) and use the market clearing condition (17). Adding $\kappa(\bar{x}-x_t)$ to account for the death and birth in the overlapping generations set-up and simplifying yields

$$\begin{aligned}
dx_t =& \kappa(\bar{x}-x_t)dt \\
&+ x_t(1-x_t)\Bigg[\left(\iota_t^* - \delta + \mu_{p,t} + \sigma_K\sigma_{p,t} - r_t - (\sigma_K+\sigma_{p,t})^2\right)\left(y_t\pi_{a,t} + (1-y_t)\pi_{b,t} - \pi_{c,t}\right) \\
&\quad - \left(y_t c_{a,t} + (1-y_t)c_{b,t} - c_{c,t}\right) \\
&\quad + \left(y_t\pi_{a,t}\frac{A_a - \iota_t^* - \Phi(\iota_t^*)}{p_t} + (1-y_t)\pi_{b,t}\frac{A_b - \iota_t^* - \Phi(\iota_t^*)}{p_t} - \pi_{c,t}\frac{A_c - \iota_t^* - \Phi(\iota_t^*)}{p_t} \right) \Bigg] dt \\
&+ x_t(1-x_t)\left[y_t\pi_{a,t} + (1-y_t)\pi_{b,t} - \pi_{c,t}\right](\sigma_K+\sigma_{p,t})dZ_t,
\end{aligned} \tag{55}$$

We calculate the dynamics of the wealth share within the intermediaries using Ito's Lemma:

$$
\begin{aligned}
dy_t =& d\frac{W_{a,t}}{W_{a,t}+W_{b,t}} \\
=& \frac{1}{W_{a,t}+W_{b,t}} dW_{a,t} - \frac{W_{a,t}}{(W_{a,t}+W_{b,t})^2} d(W_{a,t}+W_{b,t}) \\
& - \frac{1}{(W_{a,t}+W_{b,t})^2} dW_{a,t} d(W_{a,t}+W_{b,t}) + \tfrac{1}{2} \frac{2W_{a,t}}{(W_{a,t}+W_{b,t})^3} \left(d(W_{a,t}+W_{b,t})\right)^2
\end{aligned}
$$

Similar to the dynamics of x_t above, we plug in the dynamics of the wealth (Equation (6)) and the market clearing condition for capital (Equation (17)). To account for the overlapping generations, we have to add the birth and death rates $\kappa(\bar{y} - y_t)$. Then, simplifying yields

$$
\begin{aligned}
dy_t =& \kappa(\bar{y} - y_t) dt \\
& + y_t(1-y_t) \Bigg[\big((\pi_{a,t} - \pi_{b,t})(\iota_t - \delta + \mu_p + \sigma_K \sigma_{p,t} - r_t) \\
& \quad - (\pi_{a,t} - \pi_{b,t})(y\pi_{a,t} + (1-y)\pi_{b,t})(\sigma_K + \sigma_{p,t})^2 \big) \\
& \quad + \left(\pi_{a,t} \frac{A_a - \iota - \Phi(\iota)}{p_t} - \pi_{b,t} \frac{A_b - \iota - \Phi(\iota)}{p_t} \right) + c_b - c_a \Bigg] dt \\
& + y_t(1-y_t)(\pi_{a,t} - \pi_{b,t})(\sigma_K + \sigma_{p,t}) dZ_t
\end{aligned}
$$

D.3 Derivation of the HJB

The HJB-approach implies to solve (6)

$$
\begin{aligned}
0 =& \max_{c_i,\pi_i} \left\{ f(C_i, V_i) + E(dV_i) \right\} \\
=& \max_{c_i,\pi_i} \Bigg\{ f(C_i, V_i) + \frac{\partial J}{\partial W} E\left[dW\right] + \frac{\partial V_i}{\partial x} E\left[dx\right] + \frac{\partial V_i}{\partial y} E\left[dy\right] \\
&+ \frac{1}{2}\left(\frac{\partial^2 V_i}{\partial W^2} E\left[(dW)^2\right] + \frac{\partial^2 V_i}{\partial x^2} E\left[(dx)^2\right] + \frac{\partial^2 V_i}{\partial y^2} E\left[(dy)^2\right] \right) \\
&+ \frac{\partial^2 V_i}{\partial W \partial x} E\left[dW dx\right] + \frac{\partial^2 V_i}{\partial W \partial y} E\left[dW dy\right] + \frac{\partial^2 V_i}{\partial x \partial y} E\left[dx dy\right] \Bigg\} \\
=& \max_{c_i,\pi_i} \Bigg\{ f(C_i, V_i) + \frac{\partial V_i}{\partial W} W_{i,t} \left[r_t + \pi_{i,t} \left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota - \delta + \mu_p + \sigma_K \sigma_{p,t} - r_t \right) - \frac{C_{i,t}}{W_{i,t}} \right] \\
&+ \frac{\partial V_i}{\partial x} \mu_x + \frac{\partial V_i}{\partial y} \mu_y + \frac{1}{2} \left(\frac{\partial^2 V_i}{\partial W^2} W_{i,t}^2 \pi_{i,t}^2 (\sigma_K + \sigma_{p,t})^2 + \frac{\partial^2 V_i}{\partial x^2} \sigma_x^2 + \frac{\partial^2 V_i}{\partial y^2} \sigma_y^2 \right) \\
&+ \frac{\partial^2 V_i}{\partial W \partial x} W_{i,t} \pi_{i,t} \sigma_x (\sigma_K + \sigma_{p,t}) + \frac{\partial^2 V_i}{\partial W \partial y} W_{i,t} \pi_{i,t} \sigma_y (\sigma_K + \sigma_{p,t}) + \frac{\partial^2 V_i}{\partial x \partial y} \sigma_x \sigma_y \Bigg\}
\end{aligned}
$$

We use the usual guess for the indirect utility:

$$
V_i = \frac{W_i^{1-\gamma_i}}{1-\gamma_i} G_i(x,y)^{\frac{1-\gamma_i}{1-\psi_i}} \tag{56}
$$

where $G_i(x, y, K)$ describes the Value function of agent i. Plugging this in gives

$$\begin{aligned}
\rho_i + \kappa = \max_{\pi_{i,t}, c_{i,t}} \Bigg\{ & G_{i,t} + (1 - \tfrac{1}{\psi_i}) \left(r_t + \pi_{i,t} \left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota_t^* - \delta + \mu_{p,t} + \sigma_K \sigma_{p,t} - r_t \right) - c_{i,t} \right) \\
& - \tfrac{1}{\psi_i} \left(\frac{G_{i,t,x}}{G_{i,t}} \mu_{x,t} + \frac{G_{i,t,y}}{G_{i,t}} \mu_{y,t} \right) - \tfrac{1}{2}(1 - \tfrac{1}{\psi_i}) \gamma_i \pi_{i,t}^2 (\sigma_K + \sigma_{p,t})^2 \\
& - \tfrac{1}{2} \tfrac{1}{\psi_i} \left((\tfrac{1-\gamma_i}{1-\psi_i} - 1) \frac{(G_{i,t,x})^2}{G_{i,t}^2} + \frac{G_{i,t,xx}}{G_{i,t}} \right) \sigma_{x,t}^2 \\
& - \tfrac{1}{2} \tfrac{1}{\psi_i} \left((\tfrac{1-\gamma_i}{1-\psi_i} - 1) \frac{(G_{i,t,y})^2}{G_{i,t}^2} + \frac{G_{i,t,yy}}{G_{i,t}} \right) \sigma_{y,t}^2 \\
& - \tfrac{1}{\psi_i} \left((\tfrac{1-\gamma_i}{1-\psi_i} - 1) \frac{G_{i,t,x} G_{i,t,y}}{G_{i,t}^2} + \frac{G_{i,t,xy}}{G_{i,t}} \right) \sigma_{x,t} \sigma_{y,t} \\
& - \frac{1 - \gamma_i}{\psi_i} \frac{G_{i,t,x}}{G_{i,t}} \pi_{i,t} \sigma_{x,t} (\sigma_K + \sigma_{p,t}) - \frac{1 - \gamma_i}{\psi_i} \frac{G_{i,t,y}}{G_{i,t}} \pi_{i,t} \sigma_{y,t} (\sigma_K + \sigma_{p,t}) \Bigg\}. \qquad (57)
\end{aligned}$$

The first-order condition gives the optimality of the consumption:

$$\frac{\partial f}{\partial C} = \frac{\partial V_i}{\partial W} \qquad (58)$$

Deriving the Epstein-Zinn aggregator

$$\begin{aligned}
f(C_i, V_i) &= \frac{1 - \gamma_i}{1 - \frac{1}{\psi_i}} V_i \left[\left(\frac{C_i}{[(1 - \gamma_i) V_i]^{\frac{1}{1-\gamma_i}}} \right)^{1 - \frac{1}{\psi_i}} - \rho_i \right] \qquad (59) \\
&= \frac{C_i^{1 - \frac{1}{\psi_i}}}{1 - \frac{1}{\psi_i}} [(1 - \gamma_i) V_i]^{1 - \frac{1 - \frac{1}{\psi_i}}{1 - \gamma_i}} - \frac{1 - \gamma_i}{1 - \frac{1}{\psi_i}} V_i \rho_i \qquad (60)
\end{aligned}$$

with respect to the consumption yields

$$f_C(C_i, V_i) = [(1 - \gamma_i) V_i]^{1 - \frac{1 - \frac{1}{\psi_i}}{1 - \gamma_i}} C_i^{-\frac{1}{\psi_i}}$$

and the derivative with respect to W_i of our guess in Equation (56) for the indirect utility is

$$\frac{\partial V_i}{\partial W} = W_i^{-\gamma_i} G_i(x,y)^{\frac{1-\gamma_i}{1-\psi_i}}. \tag{61}$$

Hence we can plug in everything in (58) and simplify to get the optimal consumption:

$$\left[(1-\gamma_i)\frac{W_i^{1-\gamma_i}}{1-\gamma_i} G_i(x,y)^{\frac{1-\gamma_i}{1-\psi_i}}\right]^{1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}} C_i^{-\frac{1}{\psi_i}} = W^{-\gamma_i} G_i(x,y)^{\frac{1-\gamma_i}{1-\psi_i}} \tag{62}$$

$$C_i = W_i G_i(x,y) \tag{63}$$

Hence, the value function is also the consumption, wealth ratio. The aggregator becomes

$$\begin{aligned} f(C_i,V_i) =& \frac{1}{1-\frac{1}{\psi_i}} (W_i G_i(x,y))^{1-\frac{1}{\psi_i}} W_i^{(1-\gamma_i)\left(1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}\right)} G_i^{\frac{1-\gamma_i}{1-\psi_i}\left(1-\frac{1-\frac{1}{\psi_i}}{1-\gamma_i}\right)} \\ & - \frac{\rho_i}{1-\frac{1}{\psi_i}} W_i^{1-\gamma_i} G_i(x,y)^{\frac{1-\gamma_i}{1-\psi_i}} \\ =& \frac{1}{1-\frac{1}{\psi_i}} W_i^{1-\gamma_i} G_i(x,y)^{\frac{1-\gamma_i}{1-\psi_i}} (G_i - \rho_i) \end{aligned}$$

Plugging this in and simplifying, we get

$$\begin{aligned} \rho_i + \kappa =& \max_{\pi_{i,t}} \Bigg\{ G_i + (1-\tfrac{1}{\psi_i})\left(r_t + \pi_i\left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota - \delta + \mu_p + \sigma_K\sigma_{p,t} - r_t\right) - \tfrac{C_i}{W_i}\right) \\ & - \tfrac{1}{\psi_i}\left(\frac{G_{i,x}}{G_i}\mu_x + \frac{G_{i,y}}{G_i}\mu_y\right) - \tfrac{1}{2}(1-\tfrac{1}{\psi_i})\gamma_i\pi_i^2(\sigma_K+\sigma_{p,t})^2 \\ & - \tfrac{1}{2}\tfrac{1}{\psi_i}\left((\tfrac{1-\gamma_i}{1-\psi_i}-1)\frac{(G_{i,x})^2}{G_i^2} + \frac{G_{i,xx}}{G_i}\right)\sigma_x^2 - \tfrac{1}{2}\tfrac{1}{\psi_i}\left((\tfrac{1-\gamma_i}{1-\psi_i}-1)\frac{(G_{i,y})^2}{G_i^2} + \frac{G_{i,yy}}{G_i}\right)\sigma_y^2 \\ & - \tfrac{1}{\psi_i}\left((\tfrac{1-\gamma_i}{1-\psi_i}-1)\frac{G_{i,x}G_{i,y}}{G_i^2} + \frac{G_{i,xy}}{G_i}\right)\sigma_x\sigma_y \\ & - \frac{1-\gamma_i}{\psi_i}\frac{G_{i,x}}{G_i}\pi_i\sigma_x(\sigma_K+\sigma_{p,t}) - \frac{1-\gamma_i}{\psi_i}\frac{G_{i,y}}{G_i}\pi_i\sigma_y(\sigma_K+\sigma_{p,t}) \Bigg\} \end{aligned} \tag{64}$$

Deriving this equation with respect to $\pi_{i,t}$ and solving for that unknown gives the first-order condition for the portfolio weights:

$$\pi_{i,t} = \frac{1}{\gamma_i} \frac{\left(\frac{A_i - \iota - \Phi(\iota)}{p_t} + \iota - \delta + \mu_p + \sigma_K \sigma_t^p - r_t\right)}{(\sigma_K + \sigma_t^p)^2} + \frac{1}{\gamma_i} \frac{1-\gamma_i}{1-\psi_i} \left(\frac{G_{i,x}}{G_i} \frac{(\sigma_K + \sigma_t^p)\sigma_x}{(\sigma_K + \sigma_t^p)^2} + \frac{G_{i,y}}{G_i} \frac{(\sigma_K + \sigma_t^p)\sigma_y}{(\sigma_K + \sigma_t^p)^2} \right) \tag{65}$$

E Solution Algorithm

State Variables. Due to a separation of wealth approach, we can restrict the set of state variables to x_t and y_t.[18] In other words, the Value Function G_i of each agents only depends on x_t and y_t.

Auxiliary State Variable. However, a problem is that the dynamics of x_t and y_t depend on the unknown price function (through $\mu_{p,t}$ and $\sigma_{p,t}$) and the dynamics of the price function depend on x_t and y_t. This is why we will guess a price function and update the guess while simultaneously updating the guess for the value functions G_i until the program converges to the general equilibrium. Therefore, the price function $p_t(x_t, y_t)$ will serve as an auxiliary state variable.

Stationary Solution. Due to the infinite time horizon, we will solve for a stationary solution so that we can drop the time dependency.

Projection Algorithm. We opt for a projection algorithm and approximate the value functions $G_a(x,y), G_b(x,y), G_c(x,y)$ and the price function $p(x,y)$ with Chebyshev polynomials. Thus, we use

$$G_i(x,y) = \sum_{k=1}^{N} \sum_{j=1}^{N} c_{k,j}^{(i)} T_k(x) T_j(y) \qquad \text{and} \qquad p(x,y) = \sum_{k=1}^{N} \sum_{j=1}^{N} \theta_{k,j} T_k(x) T_j(y),$$

where $T_k(\cdot)$ is the k-th Chebyshev polynomial of the first kind. The advantage of this functional form is that we can directly compute all derivatives. The goal is to solve for the unknown coefficients $c_{k,j}^{(i)}$ and $\theta_{k,j}$ that solve the model at a certain set of points.

We use collocation so that there are as many gridpoints as there are unknown coefficients. The

[18] This is also shown in the derivation of the HJB equation in the appendix.

collocation points are given by a simple meshgrid. Firstly, we compute the Chebyshev nodes scaled into the unit interval for both x and y individually. The meshgrid is then simply constructed by the mesh of the individual grids for x and y.

Optimality Conditions The coefficients $c_{k,j}^{(i)}$ for the value function have to satisfy the HJB's (Equation (22)). The price has to satisfy the Goods Market Clearing condition (Equation (19)).

Initialization. The initial coefficients $c_{k,j}^{(i)}$ are set to ρ_i for $k = j = 1$ and zero otherwise. The price coefficients are initialized as 1 for $k = j = 1$ and zero otherwise.

Overview Numerical Solution. The model is solved iteratively in MatLab with the *fsolve*-function until the coefficients $c_{k,j}^{(i)}$ and $\theta_{k,j}$ satisfy the optimality conditions for the value functions G_i and the price function p_t (see above). Each iteration is as follows

1. Take the price and its derivatives from the previous iteration as given. Calculate the optimal investment rate ι_t^* as in equation (13).

2. Take the coefficients for the value functions as given from the previous iteration. Evaluate the value functions $G_i(x, y)$ and its derivatives on the gridpoints.

3. The next step is to solve for the optimal portfolio weights by using the Matlab function *fsolve*. As initialization we use $\pi_{a,t} = \pi_{b,t} = \pi_{c,t} = (\mu_{p,t} - r_t) = 1$. For $\pi_{b,t}$ and $\pi_{c,t}$, we consider the exponential of the input to ensure that the values are always positive.

   With the initial guesses for $\pi_{a,t}, \pi_{b,t}, \pi_{c,t}$ and $(\mu_{p,t} - r_t)$, we calculate $\sigma_{p,t}$ by analytically solving Equation (49) with the expressions for $\sigma_{x,t}$ and $\sigma_{y,t}$ from Equations (47) and (48). This gives

$$\sigma_{p,t} = \frac{\frac{p_x}{p} x(y\pi_a + (1-y)\pi_b - 1) + \frac{p_y}{p} y(1-y)(\pi_a - \pi_b)}{1 - \left(\frac{p_x}{p} x(y\pi_a + (1-y)\pi_b - 1) + \frac{p_y}{p} y(1-y)(\pi_a - \pi_b)\right)} \sigma_K \tag{66}$$

   We take the first-order conditions (54) for each investor and the market clearing for capital in Equation (17) with the expressions for $\sigma_{x,t}$ and $\sigma_{y,t}$ from Equations (47) and (48). This gives the following system of five equations with

$$\pi_{c,t} = \frac{1}{\gamma_c}\frac{(\iota - \delta + \mu_p + \sigma_K\sigma_{p,t} - r_t)}{(\sigma_K + \sigma_{p,t})^2} + \frac{1}{\gamma_c}\frac{A_a - \iota - \Phi(\iota)}{p_t(\sigma_K + \sigma_{p,t})^2} + \frac{1}{\gamma_c}\frac{1-\gamma_c}{1-\psi_c}\left(\frac{G_{c,x}}{G_c}x_t\left[y\pi_{a,t} + (1-y)\pi_{b,t} - 1\right] + \frac{G_{c,y}}{G_c}y_t(1-y_t)(\pi_{a,t} - \pi_{b,t})\right) \tag{67}$$

$$\pi_{b,t} = \frac{1}{\gamma_b}\frac{(\iota - \delta + \mu_p + \sigma_K\sigma_{p,t} - r_t)}{(\sigma_K + \sigma_{p,t})^2} + \frac{1}{\gamma_b}\frac{A_b - \iota - \Phi(\iota)}{p_t(\sigma_K + \sigma_{p,t})^2} + \frac{1}{\gamma_b}\frac{1-\gamma_b}{1-\psi_b}\left(\frac{G_{b,x}}{G_b}x_t\left[y\pi_{a,t} + (1-y)\pi_{b,t} - 1\right] + \frac{G_{b,y}}{G_b}y_t(1-y_t)(\pi_{a,t} - \pi_{b,t})\right) \tag{68}$$

$$\pi_{a,t} = \frac{1}{\gamma_a}\frac{(\iota - \delta + \mu_p + \sigma_K\sigma_{p,t} - r_t)}{(\sigma_K + \sigma_{p,t})^2} + \frac{1}{\gamma_a}\frac{A_c - \iota - \Phi(\iota)}{p_t(\sigma_K + \sigma_{p,t})^2} + \frac{1}{\gamma_a}\frac{1-\gamma_a}{1-\psi_a}\left(\frac{G_{a,x}}{G_a}x_t\left[y\pi_{a,t} + (1-y)\pi_{b,t} - 1\right] + \frac{G_{a,y}}{G_a}y_t(1-y_t)(\pi_{a,t} - \pi_{b,t})\right) \tag{69}$$

$$1 = xy\pi_{a,t} + x(1-y)\pi_{b,t} + (1-x)\pi_{c,t} \tag{70}$$

with $\pi_{a,t}$, $\pi_{b,t}$, $\pi_{c,t}$ and $(\mu_{p,t} - r_t)$ as unknowns. $\sigma_{p,t}$ is given as an additional output while solving the system of equations.

4. If the restriction is considered, we check if the condition in Equation (24) holds for each gridpoint. If this is not the case for a gridpoint, we set $\pi_{a,t} = \frac{\bar{\sigma}_{FI}}{(\sigma_K + \sigma_{p,t})}$ on that combination of x and y and solve the system of equations from the previous step again without considering Equation (69).

5. With the solution from the last step, we can calculate $\mu_{x,t}$, $\sigma_{x,t}$, $\mu_{y,t}$ and $\sigma_{y,t}$ (see Equations (47) and (48)). Now, we can calculate $\mu_{p,t}$ as given in Equation (49).

6. The risk-free rate is obtained by subtracting the solution for $(\mu_{p,t} - r_t)$ from step 3 from $\mu_{p,t}$ as calculated in the previous step.

7. Calculate the violation of the optimality conditions.

F Log Utility

Motivation. This section presents the model in terms of log utility. Log utility is the most commonly used utility function in asset pricing models. The popularity of log utility is due to the fact that a separation approach can be taken so that the value function doesn't need to be solved explicitly. However, these computational gains come at a cost, as log utility suffers from the well-known drawbacks of the equity premium and risk-free rate puzzle due to the entanglement of risk aversion and the intertemporal elasticity of substitution (willingness to substitute consumption over time) while also not incorporating a preference for early resolution of uncertainty.

To emphasize the contribution of Epstein-Zin preferences, we also solve our model under log utility. Since log utility is a degenerate parameterization of Epstein-Zin, this entails that we must write a separate modified algorithm to solve the model.

HJB. Since the model is the same, we will only lay out the lifetime utility maximisation problem. This is now given by

$$V_i = \max_{\{C_{i,s},\pi_{i,s},\iota_{i,s}\}_{s\in[t,\infty)}} \int_t^\infty e^{-\rho(s-t)} log\big(C(s)\big)ds \qquad \text{s.t. } \frac{dW_{i,t}}{W_{i,t}} = \pi_{i,t}dr_{i,t}^K + (1-\pi_{i,t})r_t dt - \frac{C_{i,t}}{W_{i,t}}dt.$$

For the stationary solution, the HJB for each investor is given by

$$\begin{aligned}\rho_i J(W_i,x,y) = \max_{c_i,\pi_i}\Bigg\{ & \log(C_i) + J_W E[dW_i] + J_x E[dx] + J_y E[dy] + 0.5 J_{WW} E[(dW)^2] + 0.5 J_{xx} E[(dx)^2] \\ & + 0.5 J_{yy} E[(dy)^2] + J_{Wx} E[dW dx] + J_{Wy} E[dW dy] + J_{xy} E[dx dy]\Bigg\}.\end{aligned}$$

The *initial guess* for the HJB is to separate out the level of wealth so that we postulate

$$J_i = \frac{1}{\rho_i}\log(W_i) + G_i(x,y),$$

which then yields

$$\log(W_i) + \rho_i G_i = \max_{c_i, \pi_i} \Bigg\{ \log(C_i) + \frac{1}{\rho_i} \left(r + \pi_i \big[\mu_{r_i^K} - r \big] - \frac{C_i}{W_i} \right) + G_{i,x}\mu_x + G_{i,y}\mu_y$$
$$- 0.5 \frac{1}{\rho_i} \pi_i^2 (\sigma_K + \sigma_p)^2 + 0.5 G_{i,xx}\sigma_x^2 + 0.5 G_{i,yy}\sigma_y^2 + G_{i,xy}\sigma_x\sigma_y \Bigg\},$$

where

$$\mu_{r_i^K} = \frac{A_i - \iota_t^* - \Phi(\iota_t^*)}{p} + \iota^* - \delta + \mu_p + \sigma_K \sigma_p. \tag{71}$$

Notice that $\mu_{r_i^K}$, as in the Epstein-Zin case, depends on the unknown price function $p(x, y)$.

Optimal Policies. The first-order condition for *optimal consumption* implies

$$C_i^* = \rho_i W_i, \tag{72}$$

so that agents always consume a constant fraction of wealth.

The first-order condition for the *optimal portfolio weight* is

$$\pi_i^* = \frac{\mu_{r_i^K} - r}{(\sigma_K + \sigma_p)^2}. \tag{73}$$

Notice that ι_t^* remains unchanged and is given by (13).

The first order conditions make the computational advantage of log-utility apparent: the optimal policies can be computed without any knowledge of the value function G_i. Both for CRRA or Epstein-Zin preferences can the individual level of wealth can be substituted out, but the optimal policies will depend explicitly on the value function.

Hence, the only unknown function to be solved for is the price function which we will also achieve numerically via the same projection algorithm.

Numerical Solution. We use the same set of gridpoints as for the Epstein-Zin case. Additionally, we set $p(x,y)$ as a Chebyshev Polynomial, i.e.

$$p(x,y) = \sum_{i=1}^{N}\sum_{j=1}^{N}\theta_{i,j}T_i(x)T_j(y). \tag{74}$$

With log-utility, we can instantly solve for the general equilibrium as the following will show.

To obtain the policies, we need to solve the system of first order conditions along with the market clearing conditions. This yields the following system of 5 equations with 5 unknowns (highlighted in color) which we solve by root-finding at all N^2 gridpoints by inserting (74).

$$\begin{aligned}
\pi_c^* &\overset{(73,71)}{=} \frac{A_c}{p(\sigma_K+\sigma_p)^2} + \frac{\iota^* - \delta + \mu_p - r + \sigma_K\sigma_p}{(\sigma_K+\sigma_p)^2} \\
\pi_b^* &\overset{(73,71)}{=} \frac{A_b}{p(\sigma_K+\sigma_p)^2} + \frac{\iota^* - \delta + \mu_p - r + \sigma_K\sigma_p}{(\sigma_K+\sigma_p)^2} \\
\pi_a^* &\overset{(73,71)}{=} \frac{A_a}{p(\sigma_K+\sigma_p)^2} + \frac{\iota^* - \delta + \mu_p - r + \sigma_K\sigma_p}{(\sigma_K+\sigma_p)^2} \\
1 &\overset{(17)}{=} xy\pi_a^* + x(1-y)\pi_b^* + (1-x)\pi_c^* \\
\sigma_p &\overset{(49,47,47)}{=} \frac{p_x}{p}\Big[x[y\pi_a^* + (1-y)\pi_b^* - 1](\sigma_K+\sigma_p)\Big] + \frac{p_y}{p}\Big[y(1-y)(\pi_a^* - \pi_b^*)(\sigma_K+\sigma_p)\Big].
\end{aligned}$$

We found it to be numerically more stable to solve for σ_p in close-form and inserting it in the other 4 equations. That is, we use

$$\sigma_p = \frac{\left[\frac{p_x}{p}\cdot x\left[y\pi_a^* + (1-y)\pi_b^* - 1\right] + \frac{p_y}{p}\cdot y(1-y)(\pi_a^* - \pi_b^*)\right]}{1 - \left(\frac{p_x}{p}\cdot x\left[y\pi_a^* + (1-y)\pi_b^* - 1\right] + \frac{p_y}{p}\cdot y(1-y)(\pi_a^* - \pi_b^*)\right)}\sigma_K.$$

to obtain a system of 4 equations with 4 unknowns (π_a^*, π_b^*, π_c^*, $\mu_p - r$).

Once this system has been solved for, it is straightforward to solve for μ_x and σ_x (47), μ_y and σ_y (48) and μ_p (49). Knowing μ_p and $\mu_p - r$ then instantly yields the riskless rate r. We then update the price function based on the goods market clearing condition (19) and iterate until the parameters $\theta_{i,t}$ of the price function converge.

G To Do

Gray things are done or did not have an impact

- Lösen des Modells mit zwei Investoren
  - Lösung für Logarithmic preferences
  - Lösung mit Risikoaversion gleich eins und IES ungleich eins
  - Lösung mit Risikoaversion ungleich eins und IES gleich eins.
  - Lösung mit Epstein-Zin

  Ziel: welcher Praeferenzparameter treibt welche Ergebnisse?

  Ziel: Was aendert sich in jedem Fall durch die Restriktion (laesst sich zweidimensional sehr viel leichter vergleichen als dreidimensional)?

- Lösung des Modells mit drei Investoren
  - Lösung für den Spezialfall, dass beide Intermediaere gleich sind (Betrachtung aller vier Fälle von oben).
    * Lösung für Logarithmic preferences
    * Lösung mit Risikoaversion gleich eins und IES ungleich eins
    * Lösung mit Risikoaversion ungleich eins und IES gleich eins.
    * Loesung mit Epstein-Zin

    Ziel: Loesungen muessen mit der Loesung fuer zwei Investoren uebereinstimmen, dient der Kontrolle der beiden Programme
  - Allgemeine Lösungen (siehe die vier Fälle von oben von oben)
    * Lösung für Logarithmic preferences
    * Lösung mit Risikoaversion gleich eins und IES ungleich eins
    * Lösung mit Risikoaversion ungleich eins und IES gleich eins.

    * Loesung mit Epstein-Zin

  Ziel: was aendert sich durch Heterogenitaet der Intermediaere

- Berechnung der Crisis probability als Wahrscheinlichkeit für Konsumwachstum kleiner null oder Outputwachstum kleiner null
- Einbezug von zustandsabhängigen Restriktionen, um dieses Element der Regulierung bewerten zu können (in Graz mehrfach nachgefragt, ohne dass ich in diese Richtung gelenkt haette)
- Benchmark ist nicht der unrestringierte Fall, sondern der restringierte Fall mit einer festgehaltenen Restriktionen
- Wie ändert sich die stationäre Verteilung durch die Restriktionen? Kann diese Änderung das Steigen der Wachstumsrate bei Restriktionen erklären (aber: wenn der unbedingte Erwartungswert mit der Restriktion groesser ist als ohne die Restriktion, warum implementiert dann nicht einfach immer die Strategie mit der Restriktion und ist – unbedingt – besser dran?)
- DGF 2025
  - Relation between our restriction and VaR-restriction
  - why is it beneficial to "'just" reduce financial risk
  - crisis are not recessions, play around with definitions
  - add jump process
  - be more clear on decomposition of effects
  - better explain calibration
  - robustness w.r.t. ψ, solve in closed form for $\psi = 1$ (Christoph Hambel)
  - in case of three agents, equalize risk aversion levels (Michael Weber) → *did not change the overall result*

Punkte von Patrick:

- Assess the impact of the restriction by comparing the drop of the growth rates. In EZ this drop is roughly 50%, whereas in log-utility this drop is only roughly 20%. Therefore, log-utility understates the impact of the restriction.
- Solve the log-utility model with $\rho = \{0.01, 0.02, \ldots, 0.1\}$.
  Assess if the model is sensitive to the discount rate (constant consumption-wealth ratio).
  - If sensitivity is confirmed, it indicates a potential source of misleading implications in models using log-utility. This is due to the difficulty in precisely calibrating ρ, making high sensitivity a detriment to the model's implications.
- Increase σ_K to roughly match a riskless interest rate of 1%. σ_K is a freely floating parameter and must be calibrated.
- Increase the spread of technology A. Since $Y_t = \overline{A}_t K_t$, increasing the spread increases the volatility of $\overline{A}_t$ and thereby increases the volatility of output Y_t (currently its volatility is too low).
- Assess whether $\gamma = 1$ or $\psi = 1$ is more responsible for the solution of log-utility. Solve EZ with $\psi = \psi_{baseline}$ and $\gamma = 1.01$ for all agents and see whether the policy functions match.
  - See the implications if only one agent has log utility. What does the model output if only the b agent has EZ, but the a agent has log-utility (two-agent-model).
- Calibrate the level of A_t such that the dividend yield is some realistic quantity.
- Plot σ_p for the different model settings and restrictions. Since the growth rate of $p_t K_t$ is the risky return (without dividends), σ_p is a crucial property. Moreover, σ_p is in σ_R which governs the portfolio volatility of the a-Agent and is thus crucial to assess the level of the restriction.
- To compare the 2D with the 3D Plots, i.e. two-agent-model with three-agent-model, in the 3D plot we can fix the xy combinations such that they are a fix wealth level of the a-Agent. Then, 3D contour lines can be compared to the 2D plots since the horizontal axis in the 2D plot is the wealth share of the a-Agent.